

## Contents

### Preface IX

<b>1</b>	<b>Interpolation</b>	<b>1</b>
1.1	Introduction	1
1.1.1	Which Model to be Adopted?	2
1.1.2	Which Points?	2
1.2	Classes for Function Interpolation	3
1.3	Polynomial Interpolation	4
1.3.1	Error in Polynomial Interpolation	6
1.4	Roots-Product Form	8
1.5	Standard Form	9
1.6	Lagrange Method	12
1.7	Newton Method	17
1.7.1	Coefficients' Evaluation	17
1.7.2	Previsions	22
1.7.3	Additional Data Point	23
1.7.4	Derivatives Evaluation	24
1.8	Neville Algorithm	26
1.9	Hermite Polynomial Interpolation	29
1.9.1	Lagrange-Type Method	30
1.9.2	Newton-Type Method	31
1.10	Interpolation with Rational Functions	33
1.10.1	Thiele's Continuous Fractions	35
1.10.2	Bulirsch–Stoer Method	39
1.11	Inverse Interpolation	42
1.12	Successive Polynomial Interpolation	44
1.12.1	Hermite Cubic Polynomials	45
1.12.2	Cubic Spline	49
1.13	Two-Dimensional Curves	54
1.14	Orthogonal Polynomials	54
1.14.1	Chebyshev Polynomials	56

<b>2</b>	<b>Fundamentals of Statistics</b>	<b>61</b>
2.1	Introduction	61
2.2	Fundamentals	62
2.3	Estimation of Expected Value	65
2.3.1	Random Selection	66
2.3.2	Arithmetic Mean	66
2.3.3	Median	66
2.3.4	Remedian	67
2.3.5	Trimmed Mean	68
2.3.6	Clever Mean	69
2.3.7	Mode	69
2.3.8	Symmetric and Nonsymmetric Distributions	69
2.4	Estimation of Variance	70
2.4.1	Use of Arithmetic Mean	70
2.4.2	Using the Median	71
2.4.3	Clever Variance	72
2.5	Estimation of Standard Deviation	74
2.5.1	Square Root of Variance	74
2.5.2	Unbiased Standard Deviation	74
2.5.3	Using the Median	74
2.5.4	Using the Sum of Absolute Errors	75
2.5.5	Minimum and Maximum Values	75
2.6	Outlier Detection	76
2.7	Relevant Probability Distributions	79
2.7.1	Binomial Distribution	79
2.7.2	Poisson Distribution	82
2.7.3	Normal (Gaussian) Distribution	83
2.7.4	t-Student Distribution	84
2.7.5	$\chi^2$ Distribution	87
2.7.6	F (Fisher) Distribution	89
2.8	Correct Meaning of Statistical Tests and Confidence Regions	91
2.9	Nonparametric Statistics	98
2.10	Conditional Probability	99
<b>3</b>	<b>Linear Regressions</b>	<b>101</b>
3.1	Introduction	101
3.2	Least Sum of Squares Method	103
3.3	Some Caveat	111
3.4	Class for Linear Regressions	114
3.5	Generalized Toolkit for Linear Problems	124
3.5.1	Data File Structure	126
3.5.2	Building a Data File	126
3.5.3	Data Visualization	128
3.6	Data Modification	128
3.7	Data Deletion	129

3.8	Preliminary Analysis	130
3.9	Multicollinearity	136
3.9.1	When Does the Multicollinearity Occur?	137
3.9.2	How Can Multicollinearity be Detected?	137
3.10	Best Model Selection	140
3.11	Principal Components	145
<b>4</b>	<b>Robust Linear Regressions</b>	<b>151</b>
4.1	Introduction	151
4.2	Some Caveat	151
4.3	Outliers and Gross Errors	152
4.3.1	When is an Outlier Generated?	154
4.3.2	How Can We Detect Outliers?	156
4.3.3	What Should be Done When Outliers are Detected?	168
4.4	Studentized Residuals	179
4.5	M-Estimators	181
4.6	Influential Observations	182
4.7	y-Outliers, X-Outliers, and F-Outliers	186
4.8	Secluded Observations	187
4.9	Robust Indices	189
4.10	Normality Condition	189
4.11	Heteroscedasticity Condition	190
<b>5</b>	<b>Linear Regression Case Studies</b>	<b>193</b>
5.1	Introduction	193
5.2	Ferrari F1's Test	193
5.3	Best Model Formulation	196
5.4	Outliers	200
5.4.1	Outliers Generated by Poor Quality Data	201
5.4.2	Outliers Originated by Inadequate Models	216
5.4.3	Outliers Generated by Inadequate Design of Experiments	224
5.4.4	Outliers Generated by Heteroscedasticity Condition	226
5.5	Best Model Selection	227
5.6	Principal Components	241
<b>6.</b>	<b>Nonlinear Regressions</b>	<b>245</b>
6.1	Nonlinear Regression Problems	245
6.2	Some Caveat	248
6.3	Parameter Evaluation	250
6.3.1	Test to Check the Robustness of a Minimization Program	252
6.4	BzzNonLinearRegression Class	253
6.5	Nonalgebraic Constraints	259
6.6	Algorithms for Outlier Detection	261
6.7	Correlations Among Model Parameters	263
6.8	Preventative Model Analysis	264

6.9	Model Discrimination	267
6.10	Model Collection and Model Selection	272
<b>7</b>	<b>Nonlinear Regression Case Studies</b>	<b>275</b>
7.1	Introduction	275
7.2	One Dependent Variable with Constant Variance	278
7.3	Multicubic Piecewise Models	322
7.4	One Dependent Variable and Nonconstant Variance	331
7.5	More Dependent Variables and Constant Variance	337
7.6	More Dependent Variables and Nonconstant Variance	341
7.7	Model Consisting of Ordinary Differential Equations	343
7.8	Model Consisting of Differential Algebraic Equations	352
7.9	Analysis of Alternative Models	356
7.10	Independent Variables Subject to Experimental Error	362
7.11	Variables with Missing Experiments	369
7.12	Outliers	370
7.13	Independent Variables Subject to Experimental Error and Model with Outliers	374
<b>8</b>	<b>Reasonable Design of Experiments</b>	<b>377</b>
8.1	Introduction	377
8.2	Preliminary Experiments	378
8.3	Using Models to Suggest New Experiments	380
8.4	New Experiments to Improve the Parameter Estimation	381
8.5	Model Selection: The Bayesian Approach	387
8.6	New Experiments for Model Discrimination	389
8.7	Criterion Used in BzzNonLinearRegression Class to Generate New Experiments	389
	<b>References</b>	<b>405</b>
	<b>Appendix A: Mixed-Language: Fortran and C++</b>	<b>409</b>
	<b>Appendix B: Basic Requirements for Using the BzzMath Library</b>	<b>417</b>
	<b>Appendix C: Copyrights</b>	<b>421</b>
	<b>Index</b>	<b>423</b>