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Motion of Particles and Heat Exchange in Homogeneous Isotropic Turbulence

The simplest and most extensively studied type of turbulence is statistically homogeneous isotropic turbulence of an incompressible fluid. Consequently, any new models aiming to describe turbulent transfer of momentum or heat should be tested against the case of isotropic turbulence. It is evident that the notion of isotropic turbulence as applied to large-scale turbulence represents a mathematical idealization because such flows are not occurring in nature or in technical devices. Nevertheless, it is well known that small-scale fields of velocity and temperature at large Reynolds numbers can be thought of as more or less homogeneous and isotropic. Following Monin and Yaglom (1975), we call them locally isotropic. Small-scale vortex structures are responsible for dissipation of turbulent energy and play a key role in the accumulation (clustering) of particles in a turbulent flow. Study of isotropic turbulence is thus of fundamental importance for both single-phase and two-phase flows.

1.1
Characteristics of Homogeneous Isotropic Turbulence

The present section lays out the key characteristics of homogeneous isotropic turbulence in an incompressible fluid, which is a prerequisite for the subsequent discussion of statistical behavior of particles. In the course of this presentation, we shall be using both Lagrangian and Eulerian properties.

Lagrangian correlations describe the connections between velocities and other characteristics of turbulence at various points along the mechanical trajectories of fluid elements (fluid particles). A Lagrangian single-particle (single-point) correlation moment of velocity fluctuations in the fluid is determined by

$$B_{L\,ij}(\tau) = \langle u'_i(\mathbf{x},t)u'_j(\mathbf{R}(t-\tau),\,t-\tau)|\mathbf{R}(t)=\mathbf{x}\rangle = \langle u'_i u'_j\rangle \Psi_L(\tau), \quad \langle u'_i u'_j\rangle = u'^2\delta_{ij}, \tag{1.1}$$

where $\mathbf{R}$ is the position vector of a fluid particle, $\Psi_L(\tau)$ is the Lagrangian autocorrelation function, $\langle u'_i u'_j\rangle$ is the Reynolds stress tensor of the fluid phase, and $u'^2 \equiv \langle u'_k u'_k\rangle/3$ is the intensity of velocity fluctuations in the fluid. Here and

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afterwards, angle brackets will indicate the averaging over the ensemble of turbulent fields of velocity and temperature of the carrier fluid.

The turbulent diffusion tensor of fluid particles is expressed through the Lagrangian correlation moment as

$$D_{ij}(\tau) = \int_0^{\tau} B_{L\,ij}(\xi)d\xi.$$

At large values of time, the diffusion tensor obeys the asymptotic relation

$$D_{ij} = D_t\delta_{ij}, \quad D_t = u'^2 T_L, \tag{1.2}$$

where D_t is turbulent diffusivity of an inertialess impurity and $T_L \equiv \int_0^{\infty} \Psi_L(\tau)d\tau$ is the Lagrangian integral time scale.

In scientific literature, the most commonly used approximation for the autocorrelation function is an exponential dependence of the form

$$\Psi_L(\tau) = \exp\left(-\frac{\tau}{T_L}\right). \tag{1.3}$$

Formula (1.3) is in good agreement with experimental data and with the DNS results at relatively high Reynolds numbers, except for the region of small values of τ. The behavior of the autocorrelation function (1.3) in the vicinity of $\tau = 0$ is incorrect, since $\Psi'_L(0) \neq 0$.

In isotropic turbulence, the Lagrangian integral scale T_L may be expressed in terms of kinetic energy of turbulence $k = \langle u'_n u'_n \rangle/2$ and its dissipation rate ε. This is accomplished by the relation $T_L = 4k/3C_0\varepsilon$, where C_0 is Kolmogorov's constant which, generally speaking, depends on the Reynolds number Re but assumes a constant value $C_{0\infty}$ when *Re* is large. The DNS results shown on Figure 1.1 suggest that Kolmogorov's constant can be approximated as

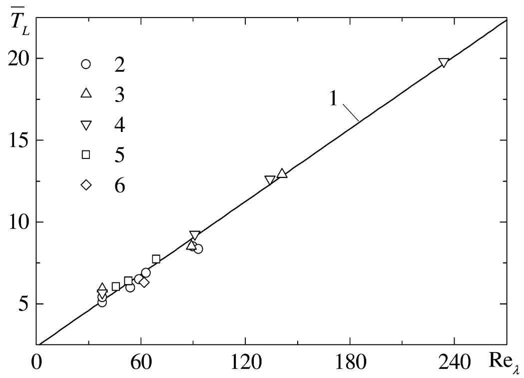

Figure 1.1 Dependence of the Lagrangian integral scale on the Reynolds number: 1 – formula (1.4), 2 – Yeung and Pope (1989), 3 – Yeung (1997), 4 – Yeung (2001), 5 – Février *et al.* (2001), 6 – Mazzitelli and Lohse (2004).

$$C_0 = \frac{C_{0\infty}\mathrm{Re}_\lambda}{\mathrm{Re}_\lambda + C_1}, \quad C_{0\infty} = 7, \quad C_1 = 32,$$

and the Lagrangian integral scale of turbulence divided by the Kolmogorov integral microscale of turbulence depends linearly on the Reynolds number:

$$T_L = \frac{T_L}{\tau_k} = \frac{2(\mathrm{Re}_\lambda + C_1)}{15^{1/2} C_{0\infty}}. \tag{1.4}$$

Following Sawford (1991), we take the asymptotic value of Kolmogorov's constant at $\mathrm{Re}_\lambda \to \infty$ equal to 7. In the above-listed formulas, $\mathrm{Re}_\lambda \equiv (15u'^4/\varepsilon\nu)^{1/2}$ is the Reynolds number calculated for the Taylor microscale; $\tau_k \equiv (\nu/\varepsilon)^{1/2}$ is the Kolvogorov time microscale; and ν is the kinematic viscosity coefficient of the fluid.

In order to describe Ψ_L for the entire range of τ, including the vicinity of $\tau = 0$, one can use the two-scale bi-exponential approximation (Sawford, 1991)

$$\Psi_L(\tau) = \frac{1}{2\Re}\left[(1+\Re)\exp\left(-\frac{2\tau}{(1+\Re)T_L}\right) - (1-\Re)\exp\left(-\frac{2\tau}{(1-\Re)T_L}\right)\right],$$

$$\Re = (1-2z^2)^{1/2}, \quad z = \frac{\tau_T}{T_L}, \tag{1.5}$$

where τ_T is the Taylor differential time scale

$$\tau_T = \left(-\frac{2}{\Psi_L''(0)}\right)^{1/2} = \left(\frac{2\mathrm{Re}_\lambda}{15^{1/2} a_0}\right)^{1/2} \tau_k. \tag{1.6}$$

Note that at $2z^2 > 1$, relation (1.5) may be represented as

$$\Psi_L(\tau) = \frac{1}{\aleph}\exp\left(-\frac{\tau}{z^2 T_L}\right)\left[\aleph\cos\left(\frac{\aleph\tau}{z^2 T_L}\right) + \sin\left(\frac{\aleph\tau}{z^2 T_L}\right)\right], \quad \aleph = (2z^2 - 1)^{1/2}.$$

The quantity a_0 in Eq. (1.6) represents the dimensionless amplitude of acceleration fluctuations in isotropic turbulence via the relation $\langle a_i a_j\rangle = a_0 \varepsilon^{3/2}\nu^{-1/2}\delta_{ij}$. According to the DNS data (Yeung and Pope, 1989; Vedula and Yeung, 1999; Gotoh and Fukayama, 2001) for the low and moderately high Reynolds numbers, to the experimental results of Voth *et al.* (2002) for the axial and transverse components of acceleration fluctuations, and to the experimental studies at relatively high Reynolds numbers in the 900 to 2000 range (Voth *et al.*, 1998), the dependence of a_0 on Re_λ (see Figure 1.2) can be approximated by

$$a_0 = \frac{a_{01} + a_{0\infty}\mathrm{Re}_\lambda}{a_{02} + \mathrm{Re}_\lambda}, \quad a_{01} = 11, \quad a_{02} = 205, \quad a_{0\infty} = 7. \tag{1.7}$$

Eulerian correlations express the connection between the parameters of a turbulent medium at fixed spatial points. Thus, the Eulerian space-time correlation moment of fluid velocity fluctuations is defined as follows:

$$B_{E\,ij}(\mathbf{r}, \tau) = \langle u_i'(\mathbf{x}, t) u_j'(\mathbf{x} - \mathbf{r},\ t - \tau)\rangle.$$

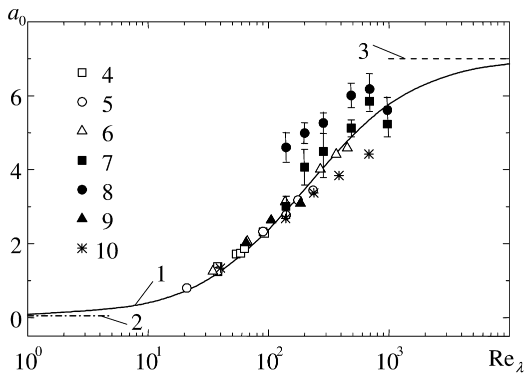

Figure 1.2 Dependence of a_0 on the Reynolds number: 1 – formula (1.7); 2 – $a_0 = 0.052$ (Vedula and Yeung, 1999); 3 – $a_0 = 7$ (Voth *et al.*, 1998); 4 – Yeung and Pope (1989); 5 – Vedula and Yeung (1999); 6 – Gotoh and Fukayama (2001); 7 – axial component (Voth *et al.*, 2002); 8 – transverse component (Voth *et al.*, 2002); 9 – Bec *et al.* (2006); 10 – Yeung *et al.* (2006).

Here and later, the Eulerian correlations are defined in a coordinate system moving with the average velocity of the medium. The most common convention is to represent the second-order space-time correlation moment as a product of spatial and temporal correlations:

$$B_{E\,ij}(\mathbf{r},\tau) = B_{ij}(\mathbf{r})\Psi_E(\tau),$$
$$B_{ij}(\mathbf{r}) = \langle u'_i(\mathbf{x},t)u'_j(\mathbf{x}-\mathbf{r},t)\rangle, \quad B_{ij}(0) = u'^2\delta_{ij}, \tag{1.8}$$

where $B_{ij}(\mathbf{r})$ is the Eulerian two-point simultaneous correlation moment and $\Psi_E(\tau)$ is the Eulerian single-point time autocorrelation function of velocity fluctuations.

In homogeneous, isotropic turbulence, any second-rank tensor can be represented as follows (Monin and Yaglom, 1975):

$$M_{ij}(\mathbf{r},\tau) = M_{nn}(r,\tau)\delta_{ij} + [M_{ll}(r,\tau) - M_{nn}(r,\tau)]\frac{r_i r_j}{r^2}, \tag{1.9}$$

where M_{ll} and M_{nn} are the longitudinal and transverse (with respect to the position vector $\mathbf{r}$) components of the tensor, and $r \equiv |\mathbf{r}|$ is the distance between the two points. For an isotropic solenoidal field such as the velocity field $\mathbf{u}(\mathbf{x})$of an incompressible fluid, there exists the following relation between the longitudinal and transverse components of this tensor:

$$M_{nn}(r,\tau) = M_{ll}(r,\tau) + \frac{r}{2}\frac{\partial M_{ll}(r,\tau)}{\partial r}. \tag{1.10}$$

According to Eq. (1.9) and Eq. (1.10), $B_{ij}(\mathbf{r})$ may be written as

$$B_{ij}(\mathbf{r}) = u'^2\left\{G(r)\delta_{ij} + [F(r) - G(r)]\frac{r_i r_j}{r^2}\right\}, \quad G(r) = F(r) + \frac{r}{2}\frac{dF(r)}{dr}, \tag{1.11}$$

where $F(r)$ and $G(r)$ are the longitudinal and transverse Eulerian spatial correlation functions.

Among the various common approximations of the Eulerian spatial and temporal correlation functions, the simplest ones are the exponential dependences

$$F(r) = \exp\left(-\frac{r}{L}\right), \quad \Psi_E(\tau) = \exp\left(-\frac{\tau}{T_E}\right), \tag{1.12}$$

characterized by integral spatial and temporal scales L and T_E. The transverse spatial correlation that corresponds to Eq. (1.12) is

$$G(r) = \left(1 - \frac{r}{2L}\right)\exp\left(-\frac{r}{L}\right). \tag{1.13}$$

As it follows from Eq. (1.13), the transverse correlation becomes negative at large values of r. The existence of the negative tail is necessitated by the requirement that mass should be conserved in accordance with the continuity equation.

To provide a description of turbulent fields that goes beyond the correlation moments, it is useful to introduce the so-called structure functions characterizing spatial and temporal increments of velocity or temperature. The Eulerian spatial structure function of the second order is defined as

$$\begin{aligned} S_{ij}(\mathbf{r}) &= \langle \Delta u'_i(\mathbf{r})\Delta u'_j(\mathbf{r})\rangle = \langle (u'_i(\mathbf{x}+\mathbf{r},t) - u'_i(\mathbf{x},t))(u'_j(\mathbf{x}+\mathbf{r},t) - u'_j(\mathbf{x},t))\rangle \\ &= 2(\langle u'_i u'_j\rangle - B_{ij}(\mathbf{r})). \end{aligned} \tag{1.14}$$

To describe the dispersion of a fluid particle pair, we introduce the Lagrangian two-particle (aka two-point) correlation moment and the Lagrangian structure function,

$$\begin{aligned} \mathbb{B}_{L\,ij}(\mathbf{r},\tau) &= \langle u'_i(\mathbf{R}_1(t),t)u'_j(\mathbf{R}_2(t-\tau),t-\tau)\rangle, \quad \mathbf{R}_1(t) = \mathbf{x}, \quad \mathbf{R}_2(t) = \mathbf{x}+\mathbf{r}, \\ \mathbb{S}_{L\,ij}(\mathbf{r},\tau) &= \langle (u'_i(\mathbf{R}_2(t),t) - u'_i(\mathbf{R}_1(t),t))(u'_j(\mathbf{R}_2(t-\tau),t-\tau) - u'_j(\mathbf{R}_1(t-\tau),t-\tau))\rangle \\ &= 2(B_{L\,ij}(\tau) - B_{L\,ij}(\mathbf{r},\tau)). \end{aligned}$$

The Lagrangian two-point correlation moment is associated with the Lagrangian single-point and Eulerian two-point correlation moments through the self-evident relations

$$\mathbb{B}_{L\,ij}(0,\tau) = B_{L\,ij}(\tau), \quad \mathbb{B}_{L\,ij}(\mathbf{r},0) = B_{L\,ij}(\mathbf{r}). \tag{1.15}$$

The relative diffusion tensor of two fluid particles may be represented as an integral of Lagrangian two-point correlations (Lundgren, 1981):

$$D^r_{ij}(\mathbf{r},\tau) = 2\int_0^\tau [B_{L\,ij}(\tau_1) - B_{L\,ij}(\mathbf{r},\tau_1)]\,d\tau_1 = \int_0^\tau \mathbb{S}_{L\,ij}(\mathbf{r},\tau_1)\,d\tau_1. \tag{1.16}$$

A Lagrangian two-point correlation, in its turn, may be written as (Zaichik and Alipchenkov, 2003)

$$\mathbb{B}_{L\,ij}(\mathbf{r},\tau) = B_{L\,ij}(\tau) + [B_{ij}(\mathbf{r}) - B_{ij}(0)]\Psi_{Lr}(\tau|r), \tag{1.17}$$

where $\Psi_{Lr}(\tau|r)$ is the Lagrangian autocorrelation function characterizing the relative motion of two particles initially separated by the distance r. It is easy to see that once we require $\Psi_{Lr}(0) = 1$, expression (1.17) obeys Eq. (1.15).

Approximation (1.17) makes it possible to represent the Lagrangian two-point structure function of velocity fluctuations in the form of a product:

$$\mathbb{S}_{L\,ij}(\mathbf{r},\tau) = S_{ij}(\mathbf{r})\Psi_{Lr}(\tau|r), \tag{1.18}$$

from which it follows that $\Psi_{Lr}(\tau|r)$ is a Lagrangian autocorrelation function of velocity fluctuation increment of fluid particles separated by a distance r.

Substitution of Eq. (1.18) into Eq. (1.16) leads to the following formula for components of the relative diffusion tensor at large values of time:

$$D^r_{ij}(\mathbf{r}) = S_{ij}(\mathbf{r})T_{Lr}, \tag{1.19}$$

where $T_{Lr} \equiv \int_0^\infty \Psi_{Lr}(\tau)d\tau$ is the two-point integral time scale characterizing the velocity fluctuation increment of the two particles.

By analogy with Eq. (1.3), the autocorrelation function of velocity fluctuation increment can be approximated by an exponential dependence:

$$\Psi_{Lr}(\tau|r) = \exp\left(-\frac{\tau}{T_{Lr}}\right). \tag{1.20}$$

Alternatively, following Eq. (1.5), we can approximate it by a two-scale bi-exponential dependence:

$$\Psi_{Lr}(\tau) = \frac{1}{2\Re_r}\left[(1+\Re_r)\exp\left(-\frac{2\tau}{(1+\Re_r)T_L}\right) - (1-\Re_r)\exp\left(-\frac{2\tau}{(1-\Re_r)T_L}\right)\right],$$
$$\Re_r = \left(1-,2z_r^2\right)^{1/2}, \quad z_r = \frac{\tau_{Tr}}{T_{Lr}}, \tag{1.21}$$

where τ_{Tr} is the Taylor differential time scale of relative velocity of two fluid particles.

Let us now consider the behavior of the structure function S_{ij}, the coefficient of relative diffusion D^r_{ij}, and the integral time scale of velocity fluctuation increment T_{Lr} in the viscous, inertial, and external spatial intervals (in that order) within the framework of Kolmogorov's similarity hypothesis for small-scale turbulence (Monin and Yaglom, 1975). This hypothesis establishes universality of small-scale turbulence in the sense that the characteristics of turbulence in the viscous and inertial intervals at large Reynolds numbers are independent of the large-scale vortex structure. Such an assumption is permissible only if we disregard the intermittency of turbulence arising from the fluctuations of the rate of turbulent energy dissipation (Monin and Yaglom, 1975; Kuznetsov and Sabel'nikov, 1990; Pope, 2000).

In the viscous interval ($r \le \eta$, where $\eta \equiv (\nu^3/\varepsilon)^{1/4}$ is the Kolmogorov spatial microscale), the first terms of Taylor's expansion of the Eulerian longitudinal and transverse structure functions are equal to (Monin and Yaglom, 1975):

$$S_{ll} = \frac{\varepsilon r^2}{15\nu}, \quad S_{nn} = \frac{2\varepsilon r^2}{15\nu}. \tag{1.22}$$

At small values of r, the difference of velocity fluctuations at two points may be represented as a linear function of the vector connecting these points, namely,

$$\Delta u_i'(\mathbf{r},\tau) = u_i'(\mathbf{x}+\mathbf{r},\tau) - u_i'(\mathbf{x},\tau) = \gamma_{ij}(\tau) r_j, \tag{1.23}$$

where $\gamma_{ij} \equiv \partial u_i'/\partial x_j$ is the velocity fluctuation gradient. In an isotropic linear field, the correlation functions of the strain and rotation tensors have the form (Girimaji and Pope, 1990; Brunk *et al.*, 1998)

$$\begin{gathered}\langle\sigma_{ik}(\mathbf{x},t)\sigma_{jn}(\mathbf{x}+\mathbf{r},t)\rangle = \frac{\varepsilon}{20\nu}\left(\delta_{ij}\delta_{kn}+\delta_{in}\delta_{jk}-\frac{2}{3}\delta_{ik}\delta_{jn}\right)\exp\left(-\frac{\tau}{\tau_\sigma}\right),\\ \sigma_{ij} = \frac{\gamma_{ij}+\gamma_{ji}}{2},\\ \langle\omega_{ik}(\mathbf{x},t)\omega_{jn}(\mathbf{x}+\mathbf{r},t)\rangle = \frac{\varepsilon}{12\nu}(\delta_{ij}\delta_{kn}-\delta_{in}\delta_{jk})\exp\left(-\frac{\tau}{\tau_\omega}\right), \quad \omega_{ij} = \frac{\gamma_{ij}-\gamma_{ji}}{2}.\end{gathered} \tag{1.24}$$

As it follows from Eq. (1.24), the strain and rotation correlation functions decrease exponentially, their respective characteristic times τ_σ and τ_ω being proportional to the Kolmogorov microscale τ_k. Expressions for the Lagrangian two-point correlation functions can be derived from Eq. (1.23) and Eq. (1.24) provided that the distribution of the distance vector between the points r_i and the distribution of the tensor of velocity fluctuation gradients γ_{ij} are statistically independent:

$$S_{L\,ll} = \frac{\varepsilon r^2}{15\nu}\exp\left(-\frac{\tau}{\tau_\sigma}\right), \quad S_{L\,nn} = \frac{\varepsilon r^2}{4\nu}\left[\frac{1}{5}\exp\left(-\frac{\tau}{\tau_\sigma}\right)+\frac{1}{3}\exp\left(-\frac{\tau}{\tau_\omega}\right)\right]. \tag{1.25}$$

By substituting Eq. (1.25) into Eq. (1.16) we obtain the longitudinal and transverse components of relative diffusion of two fluid particles of the continuum (Brunk *et al.*, 1997):

$$D_{ll}^r = \frac{\varepsilon\tau_\sigma r^2}{15\nu}, \quad D_{nn}^r = \frac{\varepsilon}{4\nu}\left(\frac{\tau_\sigma}{5}+\frac{\tau_\omega}{3}\right)r^2. \tag{1.26}$$

At $\tau_\omega = \tau_\sigma$ the relation (1.26) is consistent with the expression for the coefficient of relative diffusion derived for the viscous interval by Lundgren (1981). On the other hand, from Eq. (1.19) and Eq. (1.22) there follows

$$D_{ll}^r = \frac{\varepsilon T_{Lr} r^2}{15\nu}, \quad D_{nn}^r = \frac{2\varepsilon T_{Lr} r^2}{15\nu}. \tag{1.27}$$

Comparing Eq. (1.26) and Eq. (1.27), we see that both expressions coincide at $T_{Lr} = \tau_\omega = \tau_\sigma$. Consequently, in the viscous interval, the integral time scale of velocity fluctuation increment T_{Lr} is equal to

$$T_{Lr} = \tau_\sigma = A_1\tau_k \tag{1.28}$$

Lundgren (1981) theoretically obtained the value $\sqrt{5}$ for the constant A_1, which is in good agreement with the value 2.3 obtained by Girimaji and Pope (1990) by the DNS method.

Consider now the behavior of characteristics of turbulence in the continuous phase in the inertial interval ($\eta \ll r \ll L$), where the effect of viscosity is negligible and the particulars of large-scale convection do not play any noticeable role. The well-known similarity hypothesis proposed by Kolmogorov leads to the following self-similar representation of second order structure functions:

$$S_{ll} = C(\varepsilon r)^{2/3}, \quad S_{nn} = \frac{4}{3} C(\varepsilon r)^{2/3}, \tag{1.29}$$

where $C \approx 2.0$ according to Monin and Yaglom (1975) and Sreenivasan (1995).

It can be shown from similarity considerations that in the inertial interval, one can construct only one time scale of the order $\varepsilon^{-1/3} r^{2/3}$, so the time scale T_{Lr} should be taken as

$$T_{Lr} = A_2 \varepsilon^{-1/3} r^{2/3}, \quad A_2 = \text{const.} \tag{1.30}$$

In order to determine the constant A_2, let us recall the relations for the third-order Eulerian structure function. In the case of isotropic turbulence, any tensor of the third rank may be presented in the following form (Monin and Yaglom, 1975):

$$M_{ijk}(\mathbf{r}, \tau) = \left[M_{lll}(r, \tau) - 3M_{lnn}(r, \tau) \right] \frac{r_i r_j r_k}{r^3} + M_{lnn}(r, \tau) \left[\frac{r_i}{r} \delta_{jk} + \frac{r_j}{r} \delta_{ik} + \frac{r_k}{r} \delta_{ij} \right].$$

For an isotropic solenoidal field, the relation

$$M_{lnn}(r, \tau) = \frac{1}{6} \left[M_{lll}(r, \tau) + r \frac{\partial M_{lll}(r, \tau)}{\partial r} \right],$$

holds, indicating that the third-rank tensor $M_{ijk}(\mathbf{r}, \tau)$ is determined by just one longitudinal component $M_{lll}(r, \tau)$.

Third-order structure functions may be approximately expressed in terms of second-order structure functions via the relations (Zaichik and Alipchenkov, 2004, 2005)

$$S_{ijk} = \langle \Delta u_i' \Delta u_j' \Delta u_k' \rangle = -\frac{T_{Lr}}{3} \left(S_{in} \frac{\partial S_{jk}}{\partial r_n} + S_{jn} \frac{\partial S_{ik}}{\partial r_n} + S_{kn} \frac{\partial S_{ij}}{\partial r_n} \right). \tag{1.31}$$

Because of Eq. (1.31), the longitudinal third-order structure function of continuous phase's velocity is equal to

$$S_{lll} = -T_{Lr} S_{ll} \frac{dS_{ll}}{dr}. \tag{1.32}$$

In view of Eq. (1.29) and Eq. (1.30), it follows from Eq. (1.32) that

$$S_{lll} = -\frac{2}{3} A_2 C^2 \varepsilon r. \tag{1.33}$$

Then, taking into account the well-known Kolmogorov's relation for the inertial interval (Monin and Yaglom, 1975), we get

$$S_{lll} = -\frac{4}{5} \varepsilon r, \tag{1.34}$$

and comparing Eq. (1.33) with Eq. (1.34), finally obtain $2A_2C^2/3 = 4/5$, whence $A_2 = 0.3$ at $C = 2.0$.

At relatively large distances between the two points, fluctuations of velocity at these points are statistically independent. Thus correlation functions vanish in the external interval ($r > L$), and structure functions become equal to

$$S_{ll} = S_{nn} = 2u'^2. \tag{1.35}$$

Moreover, at large r the two-point time scale converts to the ordinary Lagrangian integral time scale

$$T_{Lr} = T_L, \tag{1.36}$$

and the tensor of relative (binary) diffusion is defined by the expression

$$D^r_{ij} = 2u'^2 T_L \delta_{ij}, \tag{1.37}$$

In other words, it equals two times the turbulent diffusion tensor of individual fluid particles (1.2).

To provide a continuous description of the longitudinal structure function of velocity fluctuations for the entire range of distances r between the two particles, we shall resort to the approximation proposed by Borgas and Yeung (2004), which combines relations (1.22), (1.29), and (1.35) for the viscous, inertial, and external spatial intervals:

$$S_{ll} = 2u'^2 \left[1 - \exp\left(-\frac{\bar{r}}{(15C)^{3/4}}\right)\right]^{4/3} \left(\frac{15^3 \bar{r}^4}{15^3 \bar{r}^4 + (2\mathrm{Re}_\lambda/C)^6}\right)^{1/6}, \quad \bar{r} = \frac{r}{\eta}. \tag{1.38}$$

The transverse structure function is expressed through the longitudinal one according to Eq. (1.10):

$$S_{nn} = S_{ll} + \frac{r}{2}\frac{\partial S_{ll}}{\partial r}. \tag{1.39}$$

Figure 1.3 presents the longitudinal and transverse structure functions calculated by formulas (1.38) and (1.39) at $C = 2$ and $\mathrm{Re}_\lambda = 61$. For comparison, we also provide structure functions obtained by Ten Cate *et al.* (2004) from the DNS with the help of the Lattice Boltzmann method. It is obvious that the longitudinal structure function tends monotonously to the limiting value of two times the intensity of velocity fluctuations. On the other hand, the behavior of the transverse structure function, which tends to $2u'^2$, is not monotonous. The maximum in the distribution of S_{nn} manifests the presence of a negative loop in the distribution of the transverse spatial correlation function $G(r)$.

The integral time scale of velocity fluctuation increment may be determined from the approximation similar to Eq. (1.38) that interpolates the relations (1.28), (1.30), and (1.36) for the corresponding characteristic intervals:

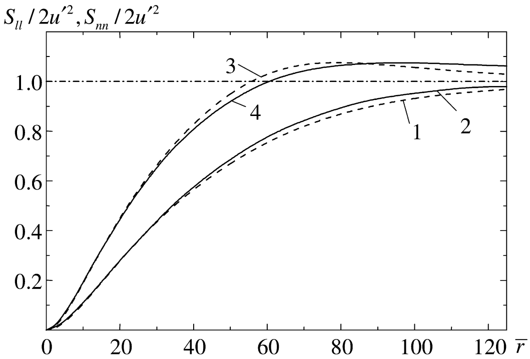

Figure 1.3 Longitudinal (1, 2) and transverse (3, 4) Eulerian structure functions of the second order: 1 – Eq. (1.38); 3 – Eqs. (1.38)–(1.39); 2, 4 – Ten Cate *et al.* (2004).

$$T_{Lr} = T_L\left[1-\exp\left(-\left(\frac{A_2}{A_1}\right)^{3/2}\overline{r}\right)\right]^{-2/3}\left(\frac{\overline{r}^4}{\overline{r}^4+(T_L/A_2)^6}\right)^{1/6}, \quad T_L = \frac{T_L}{\tau_k}. \tag{1.40}$$

The Taylor time microscale of velocity fluctuation increment τ_{Tr} can be found from the assumption that the ratio between the macro- and the microscale is independent of the distance r, that is,

$$\tau_{Tr} = \frac{\tau_T T_{Lr}}{T_L}. \tag{1.41}$$

A Lagrangian single-particle correlation moment of temperature fluctuations in the continuous phase is defined as

$$B_{Lt}(\tau) = \langle\vartheta'(\mathbf{x},t)\vartheta'(\mathbf{R}(t-\tau),t-\tau)|\mathbf{R}(t)=\mathbf{x}\rangle = \langle\vartheta'^2\rangle\Psi_{Lt}(\tau), \tag{1.42}$$

where $\Psi_{Lt}(\tau)$ is the Lagrangian autocorrelation function of temperature fluctuations, and $\langle\vartheta'^2\rangle$ is the intensity of temperature fluctuations in a turbulent fluid.

The Eulerian space-time correlation moment of temperature fluctuations can be represented as the product of spatial and temporal correlations in the manner similar to Eq. (1.8), namely,

$$B_{Et}(\mathbf{r},t) = \langle\vartheta'(\mathbf{x},t)\vartheta'(\mathbf{x}-\mathbf{r},t-\tau)\rangle = \langle\vartheta'^2\rangle F_t(\mathbf{r})\Psi_{Et}(\tau), \tag{1.43}$$

where $F_t(\mathbf{r})$ is the Eulerian two-point simultaneous correlation function of temperature fluctuations and $\Psi_{Et}(\tau)$ is the Eulerian single-point time autocorrelation function of temperature fluctuations. By analogy with Eq. (1.12), the Eulerian spatial and time correlation functions are often approximated by exponential dependences:

$$F_t(r) = \exp\left(-\frac{r}{L_t}\right), \quad \Psi_{Et}(\tau) = \exp\left(-\frac{\tau}{T_E^t}\right) \tag{1.44}$$

with the appropriate integral spatial and temporal scales L_t and T_{Et}.

1.2 Motion of a Single Particle and Heat Exchange

The subject of the present book is the behavior of small solid spherical particles in a turbulent flow. The density of particles is assumed to be much greater than that of the continuous carrier phase (fluid), and the size of particles is assumed not to exceed the Kolmogorov spatial microscale. In this case equations of motion for the particles and for the carrier flow can be written in the point-force approximation, with forces applied to the particles' centers of mass (Boivin *et al.*, 1998; Burton and Eaton, 2005). In addition, the behavior of particles in a turbulent medium under the postulated conditions is controlled primarily by the force of hydrodynamic resistance. As a rule, this force acts as the primary mechanism responsible for setting the particles in motion on the one hand, and for the opposite effect – decelerating or accelerating influence of the particles on the carrier fluid flow – on the other. Since the continuous phase density ρ_f is much smaller than the density ρ_p of particle matter, it is safe to disregard the forces arising from non-stationarity or inhomogeneity of the motion (the virtual mass effect), forces due to the acceleration or deceleration of the carrier flow (the displaced mass effect), and the Basset force, which is due to the memory effect (Maxey and Riley, 1983; Michaelides, 2003). A careful study of the influence of these forces on the turbulent motion of particles for a wide range of density ratios, $2.65 \leq \rho_p/\rho_f \leq 2650$ has been undertaken by Armenio and Fiorotto (2001), who used the DNS method. This study showed that in the entire considered range of ρ_p/ρ_f, the contribution of the virtual mass effect to the balance of forces acting on a particle is negligible, whereas the contributions of the flow acceleration effect and the memory effect may be noticeable. Nevertheless, the influence of the two corresponding forces on the dispersion of particles in a turbulent flow is still negligible compared to the resistance force, even at $\rho_p/\rho_f = O(1)$, and therefore need not be taken into account. The various effects resulting from the rotation of particles fall outside the scope of this book; bear in mind, however, that these effects can be essential for the motion of relatively large particles.

Under these assumptions, the motion of a single heavy particle is described by the equation

$$\frac{d\mathbf{R}_p}{dt} = \mathbf{v}_p, \tag{1.45}$$

$$\frac{d\mathbf{v}_p}{dt} = \frac{\mathbf{u}(\mathbf{R}_p, t) - \mathbf{v}_p}{\tau_p} + \mathbf{F}, \tag{1.46}$$

where $\mathbf{R}_p$ and $\mathbf{v}_p$ are the position and velocity of the particle and $\mathbf{u}(\mathbf{R}_p, t)$ is the velocity of the fluid at the point $\mathbf{x} = \mathbf{R}_p(t)$.

The first term on the right-hand side of Eq. (1.46) is the hydrodynamic resistance force the viscous fluid exerts on the particle, and τ_p is the characteristic time of dynamic relaxation for the particle:

$$\tau_p = \frac{\tau_{p0}}{\varphi(\mathrm{Re}_p)}, \quad \tau_{p0} = \frac{\rho_p d_p^2}{18\rho_f \nu}, \tag{1.47}$$

where τ_{p0} is the same relaxation time calculated using the Stokes approximation (that is, at $\mathrm{Re}_p \to 0$), d_p is the particle's diameter, and $\mathrm{Re}_p \equiv d_p\,|\mathbf{u} - \mathbf{v}_p|/\nu$ is the Reynolds number of the flow that goes around the particle.

The function $\varphi(\mathrm{Re}_p)$ in Eq. (1.47) describes the effect of the inertia force on hydrodynamic resistance of a spherical particle. One can find in literature a lot of formulas approximating the standard resistance curve for a spherical particle. The most commonly used one is the Schiller–Neumann approximation (Clift *et al.*, 1978)

$$\varphi(\mathrm{Re}_p) = \begin{cases} 1 + 0.15\mathrm{Re}_p^{0.687} & \text{at } \mathrm{Re}_p \le 10^3 \\ 0.11\mathrm{Re}_p/6 & \text{at } \mathrm{Re}_p > 10^3. \end{cases} \tag{1.48}$$

If the size of the particle is much smaller than the spatial microscale of turbulence, the effect of velocity fluctuations on the particle's hydrodynamic resistance is nonexistent. Bagchi and Balachandar (2003) performed the DNS for a flow that bypasses relatively large particles (whose diameter is 1.5–10 times greater than the Kolmogorov microscale) to see how turbulence affects particle resistance. It was shown that turbulent fluctuations have but a small effect on the average resistance force described by the standard correlation (1.48). But when approximation (1.48) is applied to predict the instantaneous resistance force, its accuracy falls with increase of particle size. This result proves that when studying particle interaction with turbulent eddies of the continuous phase using the point-force approximation, it is necessary to satisfy the condition $d_p < \eta$. Moreover, in the case of a flow past a large particle at high Reynolds numbers, interaction of the wake formed behind the particle with turbulent eddies of the surrounding medium assumes increased importance (Wu and Faeth, 1994, 1995 Pan and Banerjee, 1997; Bagchi and Balachandar, 2004; Legendre *et al.*, 2006). A thorough analysis of these effects is beyond the scope of present book.

The second term on the right-hand side of Eq. (1.46) denotes a force of different physical nature – for example, gravity. In near-wall flows, which are typically characterized by large gradients of all flow parameters, one needs to be aware of the lift force (Saffman force) caused by the velocity shear (Saffman, 1965, 1968). But with increase of the density ratio ρ_p/ρ_f between the disperse and continuous phases, the role of the lifting force diminishes. In the case of a non-isothermal flow, the motion of very small sub-micron particles near a heating or cooling surface may be strongly influenced by the thermophoretic force directed toward the cooler medium. Hence the symbol **F** encompasses not only the external mass force such as gravity, but also some other forces (Saffman force, thermophoretic force and so on). As opposed to the resistance force, the force **F** is considered deterministic because possible fluctuations of parameters entering the expression for **F** are usually neglected, as the effect of these fluctuations is, with rare exceptions, insufficient to make an appreciable difference.

When studying heat exchange, thermal inhomogeneity on the particle-size scale can be ignored in the majority of applied problems. Then, if we ignore heat exchange via radiation, the temperature change of a single particle is described by the equation

$$\frac{d\theta_p}{dt} = \frac{\vartheta(\mathbf{R}_p, t) - \theta_p}{\tau_t} + Q, \qquad (1.49)$$

where θ_p is particle temperature, $\vartheta(\mathbf{R}_p, t)$ – temperature of the fluid at the point $\mathbf{x} = \mathbf{R}_p\,(t)$, and τ_t – the particle's characteristic time of thermal relaxation.

The first term on the right determines the interfacial heat exchange that is taking place via conductive and convective mechanisms of heat transport in a fluid. The quantity Q denotes the intensity of heat release inside the particle (e.g., as a result of combustion). The thermal relaxation time for the particle is found from the relation

$$\tau_t = \frac{C_p \rho_p d_p^2}{6\lambda \mathrm{Nu}_p}, \qquad (1.50)$$

where C_p is heat capacity of the particle's material and is λ the coefficient of heat conductivity of the fluid.

To calculate the Nusselt number Nu_p of the flow past the particle, which enters Eq. (1.50), one can use a well-known relation (Ranz and Marshall, 1952) that is applicable for a wide range of Re_p:

$$\mathrm{Nu}_p = 2 + 0.6\mathrm{Re}_p^{1/2}\mathrm{Pr}^{1/3},$$

where Pr is the Prandtl number of the fluid.

1.3 Velocity and Temperature Correlations in a Fluid along the Inertial Particle Trajectories

The behavior of particles in a turbulent flow is governed by their interactions with turbulent eddies of the continuous phase which these particles encounter on their way. Therefore any description of statistical characteristics of the disperse phase in a turbulent fluid is critically dependent on the correlations of velocity and temperature of the fluid along inertial particle trajectories. It is obvious that these correlations coincide with the corresponding Lagrangian correlations for fluid particles in the limiting case of inertialess particles, that is, at $\tau_p \to 0$. On the other hand, in the case of highly inertial particles that show weak response to turbulent fluctuations of the continuous phase, correlations along particle trajectories should coincide with the corresponding Eulerian correlations in the fluid, which express the statistical connection between fluctuations of parameters at fixed spatial points. Hence, in order to find velocity and temperature correlations in the fluid along inertial particle trajectories, it is necessary to know the relations between Lagrangian and Eulerian correlation moments in a turbulent flow. This problem is closely linked to the problem of diffusion (dispersion) of a passive impurity (Lumley, 1962; Saffman, 1963; Kraichnan, 1964, 1970; Philip, 1967; Phythian, 1975; Lundgren and Pointin, 1976; Weinstock, 1976; Lundgren, 1981; Lee and Stone, 1983; Middleton, 1985; Squires and Eaton, 1991a; Kontomaris and Hanratty, 1993; Hesthaven *et al.*, 1995; Stepanov, 1996). Derivation of theoretical relations between Lagrangian and Eulerian correlations is facilitated by Corrsin's independence conjecture (Corrsin, 1959) – a hypothesis about independent averaging of random fields of particle displacements and Eulerian velocity fluctuations. Error analysis and the domain of applicability of Corrsin's independence conjecture is the subject of Weinstock's paper (1976). On the basis of this conjecture, Reeks (1977), Pismen and Nir (1978), and Nir and Pismen (1979) have established the correlations between fluid velocity fluctuations along particle trajectories and derived closed theoretical solutions for the problem of dispersion of heavy particles in a turbulent medium. The obtained solutions allow to describe the effect of diminishing correlativity of particle fluctuations with increase of the average velocity slip (i.e., with increase of the drift velocity of particles relative to the fluid) and the so-called crossing trajectory effect (Yudine, 1959; Csanady, 1963), and also to account properly for the influence of particle inertia on turbulent diffusion in the absence of average drift (the so-called inertia effect). Similar problems were later considered theoretically by Shraiber *et al.* (1990), Mei *et al.* (1991), Wang and Stock (1993), Mei and Adrian (1995), Stock (1996), Etasse *et al.* (1998), Pozorski and Minier (1998), Zaichik and Alipchenkov (1999), Derevich (2001), Gribova *et al.* (2003), Graham (2004). Numerical studies of particle dispersion in isotropic stationary and decaying turbulence by the DNS and LES methods were performed by Riley and Patterson (1974), Deutsch and Simonin (1991), Squires and Eaton (1991b), Yeh and Lei (1991), Elghobashi and Truesdell (1992), Mashayek *et al.* (1997), and experimental study of velocity correlations and turbulent diffusion of heavy particles is the subject of works by Snyder and Lumley (1971) and Wells and Stock (1983). The purpose of the present section is to work out a simple model that would result in suitable analytical expressions for velocity and temperature correlations in a turbulent fluid along inertial particle trajectories.

Lagrangian correlation moment of velocity fluctuations of an element of the continuous phase (i.e., fluid particle) calculated along an inertial particle trajectory has the following form (Reeks, 1977):

$$\begin{aligned} B_{Lp\,ij}(\tau) &= u'^2\Psi_{Lp\,ij}(\tau) = \langle u_i'(\mathbf{x},t)u_j'(\mathbf{R}_p(t-\tau),t-\tau)|\mathbf{R}_p(t)=\mathbf{x}\rangle \\ &= \int\langle u_i'(\mathbf{x},t)u_j'(\mathbf{x}-\mathbf{r},t-\tau)\delta(\mathbf{r}-\mathbf{s}(\tau))\rangle d\mathbf{r}, \end{aligned}$$

$$\mathbf{s}=\mathbf{S}+\mathbf{s}',\quad \mathbf{S}=\mathbf{W}\tau,\quad \mathbf{s}'=\int_0^\tau \mathbf{v}_p'(\mathbf{R}_p(\tau_1))d\tau_1. \tag{1.51}$$

Here $\Psi_{Lp\,ij}(\tau)$ is the Lagrangian autocorrelation function of fluid velocity along the particle trajectory and $\mathbf{R}_p$ is the position vector of a point on that trajectory. Displacement $\mathbf{s}$ of a particle relative to the moving fluid is the sum of two components, which arise from two independent processes. The first component is due to the drift, which is characterized by the drift velocity $\mathbf{W}$ (i.e. particle's velocity relative to the average velocity of the surrounding medium), and the second (fluctuational) component arises as a result of the particle's involvement in turbulent motion. Note that, unlike the scalar Lagrangian function Ψ_L (τ), the autocorrelation function of fluid velocity fluctuations $\Psi_{Lp\,ij}(\tau)$ is a tensor, because fluid velocity field along the particle trajectory is non-isotropic in view of the particle's average drift relative to the fluid.

In order to calculate the integral (1.51), we must employ Corrsin's hypothesis about the possibility of independent statistical averaging of random fields of particle displacements and Eulerian velocity fluctuations. In accordance with this hypothesis, we obtain the following:

$$\langle u_i'(\mathbf{x},t)u_j'(\mathbf{x}-\mathbf{r},t-\tau)\delta(\mathbf{r}-\mathbf{s}(\tau))\rangle = \langle u_i'(\mathbf{x},t)u_j'(\mathbf{x}-\mathbf{r},t-\tau)\rangle\phi(\mathbf{r},\tau),$$
$$\phi(\mathbf{r},\tau) = \langle\delta(\mathbf{r}-\mathbf{s}(\tau))\rangle, \tag{1.52}$$

where $\phi(\mathbf{r},\tau)$is the probability density of particle displacement $\mathbf{r}$ at the moment τ. The quantity $\phi(\mathbf{r},\tau)$ usually has a Gaussian distribution in either the frequency space or the coordinate space, with the variance expressed through a Lagrangian autocorrelation function. Then Eq. (1.51) becomes non-linear and implicit because it contains $\Psi_{Lp\ ij}(\tau)$ on both the left-hand side and the right-hand side. Therefore its solution can be obtained only by numerical or iterative methods. In order to avoid the iteration procedure and still obtain a simple explicit expression for $\Psi_{Lp\ ij}(\tau)$ that could be easily used in further calculations, we shall take the probability density of particle displacement in the form of a δ-function:

$$\phi(\mathbf{r},\tau) = \delta(\mathbf{r}-\mathbf{s}(\tau)). \tag{1.53}$$

Substituting Eq. (1.52) and Eq. (1.53) into Eq. (1.51) and taking into account the relations (1.8) and (1.11) for the Eulerian space-time correlation moment in isotropic turbulence, we arrive at the following expression for the Lagrangian autocorrelation function of fluid velocity fluctuations along the particle trajectory:

$$\Psi_{Lp\ ij}(\tau) = \left[G(s)\delta_{ij} + [F(s)-G(s)]\frac{\langle s_i s_j\rangle}{s^2}\right]\Psi_E(\tau),$$
$$\langle s_i s_j\rangle = W_i W_j\tau^2 + \langle s_i' s_j'\rangle,\ s = \langle s_k s_k\rangle^{1/2}. \tag{1.54}$$

The fluctuational component of particle displacement is estimated by approximate integration of the equations of motion (1.45) and (1.46):

$$\mathbf{s}' = \frac{1}{\tau_p}\int_0^{\tau}\int_0^{\tau_1}\mathbf{u}'(\mathbf{R}_p(\tau_2))\exp\left(-\frac{\tau_1-\tau_2}{\tau_p}\right)d\tau_2 d\tau_1 \approx \mathbf{u}_0'\left\{\tau+\tau_p\left[\exp\left(-\frac{\tau}{\tau_p}\right)-1\right]\right\}. \tag{1.55}$$

Let us set the characteristic value of velocity fluctuations in Eq. (1.55) equal to the root-mean-square value of fluctuational velocity u':

$$|\mathbf{u}_0'| = u'. \tag{1.56}$$

Then relations (1.55) and (1.56) give us the following:

$$\langle s_i' s_j'\rangle = \frac{u'^2\Psi^2(\tau)}{3}\delta_{ij},\ \Psi(\tau) = \tau+\tau_p\left[\exp\left(-\frac{\tau}{\tau_p}\right)-1\right]. \tag{1.57}$$

It is easily seen that approximation (1.54) together with Eq. (1.57) accounts for both the crossing trajectory effect, which is caused by the drift velocity $\mathbf{W}$, and the inertia effect, which is characterized by the particle response time τ_p. Next, for the Eulerian temporal and spatial longitudinal correlation functions in Eq. (1.54) we shall take their exponential dependences (1.12). In doing so, we must keep in mind that these single-scale approximations, are, strictly speaking, justified only in the limit of high Reynolds numbers. But they can be adapted for use at finite Reynolds numbers by taking into account the dependence of the integral scales on Re_λ. Substitution of Eq. (1.12), Eq. (1.13), and Eq. (1.57) into Eq. (1.54) yields the following expressions for the autocorrelation tensor of fluid velocity fluctuations along the particle trajectory:

$$\begin{aligned}\Psi_{Lp\,ij}(\tau) = {} & \left\{\delta_{ij} + \frac{m\left[3\gamma^2\tau^2 e_i e_j - (3\gamma^2\tau^2 + 2\psi^2(\tau))\delta_{ij}\right]}{6(\gamma^2\tau^2+\psi^2(\tau))^{1/2}T_E}\right\} \\ & \times \exp\left[-\frac{\tau + m(\gamma^2\tau^2+\psi^2(\tau))^{1/2}}{T_E}\right],\end{aligned} \tag{1.58}$$

where $m \equiv T_E u'/L$ is the structure parameter of turbulence, $\gamma \equiv W/u'$ – the drift parameter, $W \equiv |\mathbf{W}|$ – the absolute value of the drift velocity, $e_i \equiv W_i/W$ – components of the unit vector that points in the direction of the drift velocity.

In the absence of drift ($\gamma = 0$), the tensor $\Psi_{Lp\,ij}(\tau)$ becomes isotropic, and $\Psi_{Lp\,ij}(\tau)$ $\Psi_{Lp}(\tau)\delta_{ij}$, where in accordance with Eq. (1.58),

$$\Psi_{Lp}(\tau) = \left[1 - \frac{m\psi(\tau)}{3T_E}\right]\exp\left[-\frac{\tau + m\psi(\tau)}{T_E}\right]. \tag{1.59}$$

In the limiting case of inertialess particles ($\tau_p \to 0$), it follows from Eq. (1.59) that

$$\Psi_L(\tau) = \left(1 - \frac{m\tau}{3T_E}\right)\exp\left[-\frac{(1+m)\tau}{T_E}\right]. \tag{1.60}$$

Formula (1.60) represents the Lagrangian autocorrelation function of fluid particle velocity fluctuations in terms of Eulerian variables, in other words, it is characterized by the Eulerian integral time scale T_E and by the structure parameter m. From Eq. (1.60) there follows a simple relation between the Lagrangian and Eulerian temporal macroscales:

$$\frac{T_L}{T_E} = \frac{3+2m}{3(1+m)^2}, \tag{1.61}$$

which shows that the ratio of these scales, T_L/T_E, depends on structure parameter m, cannot exceed unity, tends to unity at $m \to 0$ and falls off with increase of m. Such behavior of T_L/T_E is in complete agreement with the results obtained by other authors, for example, Philip (1967), Lee and Stone (1983), Middleton (1985), Wang and Stock (1993), Stepanov (1996), Derevich (2001). Experimental data for isotropic grid turbulence give $T_L u'/L \approx 0.3 - 0.6$ (Sato and Yamamoto, 1987). The DNS results predict $T_L/T_E = 0.72 \pm 0.06$, $m \approx 1$ (Yeung and Pope, 1989) and $T_L/T_E \approx 0.75$, $m \approx 0.7$ (Mazzitelli and Lohse, 2004) for stationary turbulence, and $T_L/T_E \approx 0.82$ (Squires and Eaton, 1991a) for decaying turbulence. The method of kinematic simulation, which

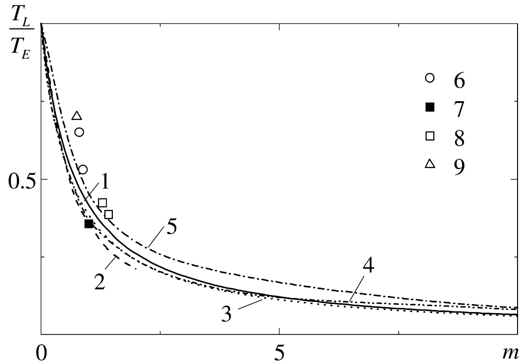

Figure 1.4 Ratio of Lagrangian and Eulerian time macroscales vs. structure parameter of turbulence: 1 – (1.61); 2 – Philip (1967); 3 – Lee and Stone (1983); 4 – Stepanov (1996); 5 – Derevich (2001), 6 – Fung *et al.* (1992); 7 – Wang and Stock (1993); 8 – Oesterlé (2004); 9 – Mazzitelli and Lohse (2004).

approximates non-stationary velocity field of the continuum by a superposition of random Fourier modes, gives $T_L/T_E = 0.53 - 1.11$, $m = 0.5 - 0.88$ (Fung *et al.*, 1992) and $T_L/T_E \approx 0.4$, $m \approx 1.3 - 1.4$ (Oesterlé, 2004). Hence the scale ratio T_L/T_E in extensively studied isotropic flows usually varies from 0.3 to 0.8 and the structure parameter is $m \approx 1$. This is why Wang and Stock, 1993 adduce the results of calculations for $m = 1$. For $m = 1$, formula (1.61) gives $T_L/T_E = 0.417$, which is slightly above the value $T_L/T_E = 0.356$ given by Wang and Stock (1993). Figure 1.4 illustrates the correlation of Eq. (1.61) with theoretical dependences as well as the results of numerical calculations by other authors. It should be noted that the inequality $T_L < T_E$ results from the fact that T_E reflects only the temporal decay of velocity fluctuation correlations in a turbulent fluid at a fixed point in space, whereas T_L takes into account both temporal and spatial decay of velocity correlations for a fluid particle in the course of its motion. Hence formula (1.61) is in good agreement with the theoretical dependences as well as with the results of numerical calculations by the authors listed in the caption for Figure 1.4.

In view of Eq. (1.61), Lagrangian autocorrelation function (1.60) takes the form

$$\Psi_L(\tau) = \left[1 - \frac{m(3+2m)\tau}{9(1+m)^2 T_L}\right] \exp\left[-\frac{(3+2m)\tau}{3(1+m)T_L}\right]. \qquad (1.62)$$

As evidenced by Figure 1.5, the influence of parameter m on the dependence $\Psi_L(\tau/T_L)$ is very weak, and for all possible values of m this dependence is only slightly different from the exponential approximation (1.3) corresponding to the limit $m \to 0$. The peculiarity of the dependence (1.62) is the existence of a negative loop (negative tail) at large values of τ/T_L, which, however, is barely noticeable at finite values of parameter m. Even for the limiting dependence

$$\Psi_L(\tau) = \left(1 - \frac{2\tau}{9T_L}\right) \exp\left(-\frac{2\tau}{3T_L}\right), \qquad (1.63)$$

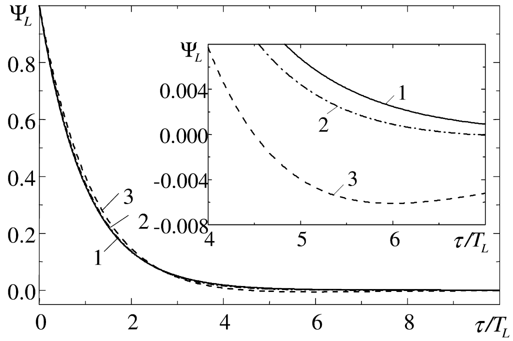

Figure 1.5 Influence of the structure parameter of turbulence on the Lagrangian autocorrelation function: 1 – $m = 0$ (1.3); 2 – Eq. (1.63) at $m = 1$; 3 – $m \to \infty$, i.e., Eq. (1.63).

at $m \to \infty$, the maximum negative value of Ψ_L is equal only to 0.0061 at $\tau/T_L = 6$. Therefore the exponential dependence (1.3) is a good approximation for the Lagrangian autocorrelation function of velocity fluctuations of fluid (inertialess) particles at all possible values of structure parameter m.

In view of expression (1.59) for the autocorrelation function, the integral scale of fluid velocity fluctuations along inertial particle trajectories in the absence of average slip (drift) of particles relative to the fluid is given by

$$T_{Lp} = \int_0^\infty \Psi_{Lp}(\tau)\, d\tau = \int_0^\infty \left[1 - \frac{m\psi(\tau)}{3T_E}\right] \exp\left[-\frac{\tau + m\psi(\tau)}{T_E}\right] d\tau. \tag{1.64}$$

The quantity T_{Lp} may be considered as the duration of particle interactions with energy-carrying turbulent eddies. It follows from Eq. (1.64) that T_{Lp} is determined by two parameters: the Stokes number $\mathrm{St}_E \equiv \tau_p/T_E$, which characterizes particle inertia with respect to the time macroscale of turbulence, and the structure parameter m. As one can see from Figure 1.6, the value of T_{Lp} increases monotonously from the Lagrangian to the Eulerian macroscale as the Stokes number St_E grows from 0 to ∞. It should be noted that the quantity $u'\psi(\tau)$ represents the effective free path of a particle undergoing fluctuational motion. Decrease of the effective free path with increase of the Stokes number explains why the correlation between turbulent characteristics of fluid particles moving along inertial particle trajectories changes from Lagrangian to Eulerian. The influence of the parameter m manifests itself through the ratio T_L/T_E. When the structure parameter is less than one ($m \le 1$), the integral (1.64) is approximated by the formula

$$T_{Lp} = T_L + (T_E - T_L) f(\mathrm{St}_E), \quad f(\mathrm{St}_E) = \frac{\mathrm{St}_E}{1 + \mathrm{St}_E} - \frac{0.9m\mathrm{St}_E^2}{(1 + \mathrm{St}_E)^2 (2 + \mathrm{St}_E)}, \tag{1.65}$$

which asymptotically approaches the limiting relations at $\mathrm{St}_E \to 0$, $\mathrm{St}_E \to \infty$ and $m \to 0$. It is easy to see that formula (1.65) gives a good approximation of Eq. (1.64)

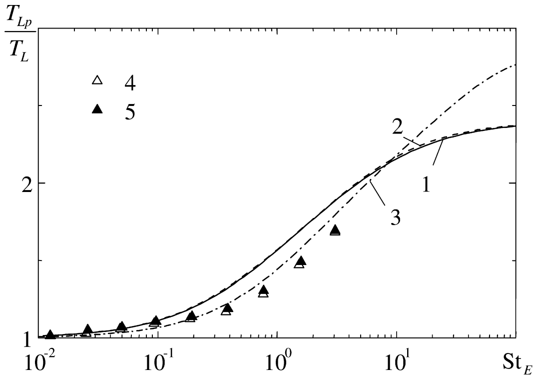

Figure 1.6 Influence of the Stokes number on the duration of particle interaction with turbulent eddies at $m=1$: 1 – (1.64); 2 – (1.65); 3 – (1.66); 4, 5 – Oesterlé (2004); 4 – $\mathrm{Re}_\lambda=38$; 5 – $\mathrm{Re}_\lambda=112$.

even at $m=1$. The dependence of T_{Lp}/T_L on Stokes number St_E,

$$T_{Lp}=\left[1-\frac{0.644}{(1+\mathrm{St}_E)^{0.4(1+0.01\mathrm{St}_E)}}\right]T_E,\qquad \frac{T_L}{T_E}=0.356, \tag{1.66}$$

which was obtained by Wang and Stock (1993) as an approximation of the general equations at $m=1$, is also shown on Figure 1.6. The figure suggests a qualitative agreement between the relations (1.64) and (1.66), even though the kinematic simulation data obtained by Oesterlé (2004) is a better match for Eq. (1.66).

The autocorrelation function (1.59) at $m=1$ and different values of St_E is plotted against time on Figure 1.7a and 1.7b, time being divided by T_E and T_{Lp}, respectively. Figure 1.7a demonstrates the growing correlativity of velocity of a fluid particle moving along a solid inertial particle trajectory as its inertia increases. Figure 1.7b makes it clear that the dependence of variables ψ_{Lp} and τ/T_{Lp} on Stokes number St_E vanishes and the exponent

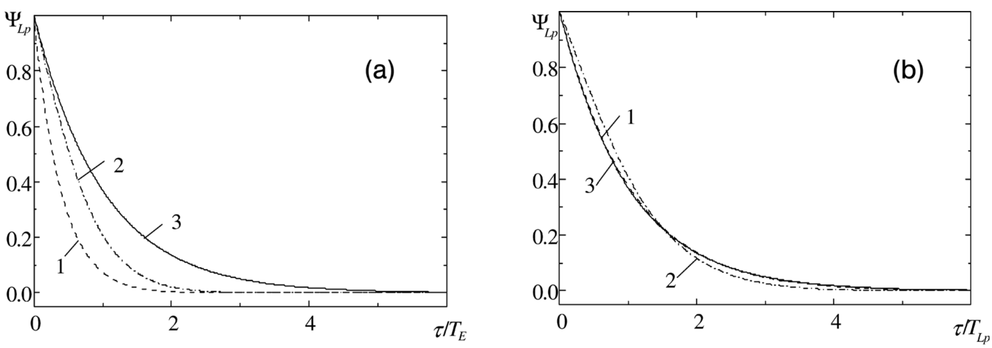

Figure 1.7 Autocorrelation function of fluid particle's velocity along an inertial particle trajectory: 1 – $\mathrm{St}_E=0$; 2 – $\mathrm{St}_E=1$; 3 – $\mathrm{St}_E=\infty$ (1.67).

$$\Psi_{Lp}(\tau) = \exp\left(-\frac{\tau}{T_{Lp}}\right) \tag{1.67}$$

corresponding to the limiting case $\mathrm{St}_E \to \infty$ is a good approximation for the dependence $\Psi_{Lp}(\tau/T_{Lp})$ even at $\mathrm{St}_E = 1$.

In the presence of particle drift, the duration of particle's interaction with energy-carrying eddies is a tensor defined as

$$T_{Lp\,ij} = \int_0^\infty \Psi_{Lp\,ij}(\tau) d\tau.$$

In accordance with Eq. (1.58), the tensors of the autocorrelation function $\Psi_{Lp\,ij}$ and the duration of particle interaction $T_{Lp\,ij}$ in isotropic turbulence may be written as

$$\Psi_{Lp\,ij}(\tau) = \begin{pmatrix} \Psi^l_{Lp} & 0 & 0 \\ 0 & \Psi^n_{Lp} & 0 \\ 0 & 0 & \Psi^n_{Lp} \end{pmatrix}, \quad T_{Lp\,ij} = \int_0^\infty \Psi_{Lp\,ij}(\tau)\, d\tau = \begin{pmatrix} T^l_{Lp} & 0 & 0 \\ 0 & T^n_{Lp} & 0 \\ 0 & 0 & T^n_{Lp} \end{pmatrix}, \tag{1.68}$$

where the top indexes l and n denote parallel (longitudinal) and transverse components of these tensors (here the terms "parallel" and "transverse" mean "with respect to the vector of the average drift velocity of particles). Parallel and transverse components of the tensor $\mathbf{T}_{Lp}$ are written as

$$T^l_{Lp} = \int_0^\infty \left[1 - \frac{m\psi^2(\tau)}{3(\gamma^2\tau^2 + \psi^2(\tau))^{1/2} T_E}\right] \exp\left[-\frac{\tau + m(\gamma^2\tau^2 + \psi^2(\tau))^{1/2}}{T_E}\right] d\tau,$$

$$T^n_{Lp} = \int_0^\infty \left[1 - \frac{m(3\gamma^2\tau^2 + 2\psi^2(\tau))}{6(\gamma^2\tau^2 + \psi^2(\tau))^{1/2} T_E}\right] \exp\left[-\frac{\tau + m(\gamma^2\tau^2 + \psi^2(\tau))^{1/2}}{T_E}\right] d\tau. \tag{1.69}$$

As evidenced by Eq. (1.58) and Eq. (1.69), the reduction of velocity correlations in a turbulent fluid resulting from the crossing trajectory effect is greater in the longitudinal than in the transverse direction. This fact was first noticed by Csanady (1963) and was called "the continuity effect", since it stems from the continuity equation (1.11) connecting the longitudinal and transverse spatial Eulerian correlation functions. In the limits of small and large values of the Stokes number St_E Eq. (1.69) gives rise to the asymptotic relations

$$T^l_{Lp}(\mathrm{St}_E \to 0) = \frac{3\Im + m(2 + 3\gamma^2)}{3\Im(1 + m\Im)^2} T_E, \; T^n_{Lp}(\mathrm{St}_E \to 0) = \frac{6\Im + m(4 + 3\gamma^2)}{6\Im(1 + m\Im)^2} T_E,$$

$$\Im = (1 + \gamma^2)^{1/2}, \quad T^l_{Lp}(\mathrm{St}_E \to \infty) = \frac{T_E}{1 + m\gamma}, \quad T^n_{Lp}(\mathrm{St}_E \to \infty) = \frac{2 + m\gamma}{2(1 + m\gamma)^2} T_E. \tag{1.70}$$

In another limiting case, when the crossing trajectory effect assumes a leading role ($\gamma \to \infty$), that is, when the drift velocity exceeds the intensity of turbulent fluctuations by a sufficient amount, Eq. (1.69) leads to the well-known relations (Yudine, 1959; Csanady, 1963)

$$T_{Lp}^{l} = \frac{L}{W}, \quad T_{Lp}^{n} = \frac{L}{2W}, \quad \frac{T_{Lp}^{l}}{T_{Lp}^{n}} = 2.$$

To make an approximate estimation of the duration of particle's interaction with turbulent eddies while taking into account the crossing trajectory effect and the particle inertia effect, we can use interpolations that are based on Eq. (1.65) and Eq. (1.70):

$$T_{Lp}^{l,n} = T_{Lp}^{l,n}(\mathrm{St}_E \to 0) + \left[T_{Lp}^{l,n}(\mathrm{St}_E \to \infty) - T_{Lp}^{l,n}(\mathrm{St}_E \to 0)\right] f(\mathrm{St}_E). \quad (1.71)$$

Formula (1.71) approximates the integrals (1.69) in the parameter range $m \leq 1$, $0 \leq \mathrm{St}_E < \infty$, $0 \leq \gamma < \infty$ with the error of no more than 5%.

In an effort to incorporate both effects – the inertia effect and the crossing trajectory effect – into their model, Wang and Stock (1993) employed the assumption that the parallel component of the autocorrelation function is constant over the ellipse

$$\frac{\tau^2}{T_{Lp}^2} + \frac{W^2\tau^2}{L^2} = \text{const},$$

where T_{Lp} denotes the duration of particle interaction with turbulence in the absence of drift ($\gamma = 0$). This *a priori* suggestion enables a smooth transition from a flow with no drift to a flow where drift is a major factor as γ goes from 0 to ∞, and allows to obtain the following simple formulas for the parallel and transverse components of the tensor $\mathbf{T}_{Lp}$:

$$T_{Lp}^{l} = \frac{T_{Lp}}{\left(1 + m_T^2\gamma^2\right)^{1/2}}, \quad T_{Lp}^{n} = \frac{2\left(1 + m_T^2\gamma^2\right)^{1/2} - m_T\gamma}{2\left(1 + m_T^2\gamma^2\right)} T_{Lp}, \quad m_T = \frac{T_{Lp}}{T_E} m, \quad (1.72)$$

where T_{Lp} is determined by Eq. (1.66).

Using Csanady's (1963) relations for the diffusion coefficient of particles in isotropic turbulence in the presence of gravity as their starting point, Deutsch and Simonin (1991) proposed the following simple formulas for the parallel and transverse components:

$$T_{Lp}^{l} = \frac{T_L}{(1 + \beta^2\gamma^2)^{1/2}}, \quad T_{Lp}^{n} = \frac{T_L}{(1 + 4\beta^2\gamma^2)^{1/2}}, \quad \beta = \frac{T_L u'}{L}. \quad (1.73)$$

It is obvious that the dependences (1.73) include the crossing trajectory and continuity effects but ignore the inertia effect. By comparing experimental measurements (Wells and Stock, 1983) with their own numerical results, Deutsch and Simonin (1991) concluded that $\beta^2 = 0.45$.

Figure 1.8 shows how the crossing trajectory effect influences the duration of particle interaction with turbulent eddies according to the relations (1.69) and (1.72) for $m = 1$. For comparison, the same figure also presents the results of direct stochastic trajectory simulation of particle dynamics by Deutsch and Simonin, who employed the LES method to calculate the turbulent characteristics of the carrier flow (Deutsch and Simonin, 1991). In this work, calculations were carried out for particles

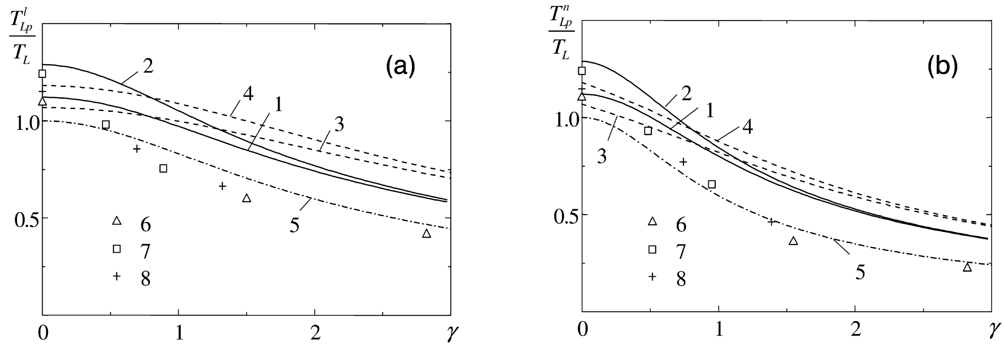

Figure 1.8 Influence of the average slip on the duration of particle interaction with turbulent eddies in parallel (a) and transverse (b) directions (with respect to the drift velocity): 1, 2 – (1.69); 3, 4 – (1.72); 1, 3 – $St_E = 0.1$; 2, 4 – $St_E = 0.3$; 5 – (1.73); 6–8 – Deutsch and Simonin (1991).

of diameters $d_p = 45$, 57, and 90 μm, the density ratio between the particle material and the carrier fluid being equal to $\rho_p/\rho_f = 2000$. As one can see from the graphs of $T^{l,n}_{Lp}$ versus the drift parameter γ shown on Figure 1.8, the dependences (1.69) and (1.72) are roughly similar, though not as close to each other as the corresponding dependences obtained by Deutsch and Simonin, whose results for relatively large values of γ slightly favor the Eq. (1.69). It can also be seen that with increase of the average slip, the influence of particle inertia, that is, of the Stokes number, on T_{Lp} is eliminated, and $T^{l,n}_{Lp}/T_L$ is governed chiefly by γ. Also shown on Figure 1.8 are the dependencies (1.73), which match the numerical data quite well except for the regions of small γ – as one might have expected, because these dependences do not take into account the inertia effect and the influence of St_E on $T^{l,n}_{Lp}$.

Figure 1.9a and 1.9b describe the autocorrelation functions Ψ^l_{Lp} and Ψ^n_{Lp} calculated by the formula (1.58) at $m = St_E = 1$ for different values of drift parameter γ.

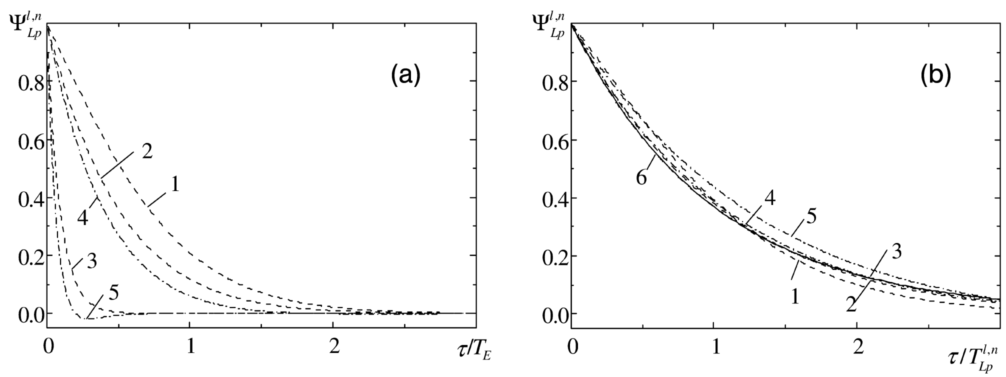

Figure 1.9 Autocorrelation functions at different drift parameter values: 1 – $\gamma = 0$; 2, 3 – Ψ^l_{Lp}; 4, 5 – Ψ^n_{Lp}; 2, 4 – $\gamma = 1$; 3, 5 – $\gamma = 10$; 6 – (1.74).

Figure 1.9a shows the reduction of velocity correlativity of fluid particles moving along inertial particle trajectories with increase of the drift parameter, which is just the physical manifestation of the crossing trajectory effect. One can easily notice the growing difference between the parallel and transverse components of correlation functions as the drift parameter gets larger.

When the drift parameter and the Stokes number both tend to infinity ($\gamma \to \infty$, $\mathrm{St}_E \to \infty$), the autocorrelation functions tend to the asymptotic dependences

$$\Psi^{\ell}_{Lp}(\tau) = F\left(\frac{W\tau}{L}\right)\Psi_E\left(\frac{\tau}{T_E}\right), \quad \Psi^{n}_{Lp}(\tau) = G\left(\frac{W\tau}{L}\right)\Psi_E\left(\frac{\tau}{T_E}\right), \tag{1.74}$$

which express fluid velocity correlations along an inertial particle trajectory as the product of Eulerian spatial and temporal velocity correlation functions of the fluid. The negative loop in the transverse autocorrelation function becomes more visible with increase of the drift parameter. This fact, which is consistent with formula (1.74), has been discussed by Mei *et al.* (1991) and Squires and Eaton (1991b). Figure 1.9b illustrates the diminishing role of parameter γ as well as the diminishing difference between Ψ^{l}_{Lp} and Ψ^{n}_{Lp} when the times entering these two parameters are scaled by their corresponding integral scales. Having said that, the distinction between the longitudinal and transverse components is still felt very clearly. Figure 1.9b also shows the exponential approximation of autocorrelations,

$$\Psi_{Lp\,ij}(\tau) = \exp\left(-\tau T^{-1}_{Lp\,ij}\right). \tag{1.75}$$

It is evident from Figure 1.9b that the dependence $\Psi^{l}_{Lp}(\tau/T^{l}_{Lp})$is accurately described by the exponential function (1.75). On the other hand, the deviation of $\Psi^{n}_{Lp}(\tau/T^{n}_{Lp})$ from the exponential function is more noticeable.

By using stochastic simulation, Berlemont *et al.* (1995) found that the duration of interaction between the turbulence and the particles increases with frequency of interparticle collisions. At the first glance this result seems counterintuitive because collisions should reduce the correlation between the motion of inertial particles and that of the fluid and thereby shorten the interaction time. But in reality, the effect of increasing T_{Lp} with increase of collision frequency has the same explanation as the particle inertia effect and is related to the inequality $T_E > T_L$.

To understand the ultimate effect of collisions, we should first see how they change the effective mean free path $u'\psi(\tau)$ of a particle undergoing fluctuational motion and then make the corresponding adjustments to Eq. (1.58). Suppose that collisions cause particles to lose all memory about their preceding involvement in fluctuational motion of the carrier flow and thereby – in the turbulent flow. The end result is that whenever a collision occurs, the interaction of the particle with turbulent eddies starts again from the scratch. In this case it follows from the approximate solution of the equations of motion (1.45) and (1.46) that

$$\psi(\tau) = \tau + n\tau_p\left[\exp\left(-\frac{\tau_c}{\tau_p}\right) - 1\right] + \tau_p\left[\exp\left(-\frac{\tau - n\tau_c}{\tau_p}\right) - 1\right],$$
$$n\tau_c < \tau < (n+1)\tau_c, \; n = 0, 1, 2, \ldots, \tag{1.76}$$

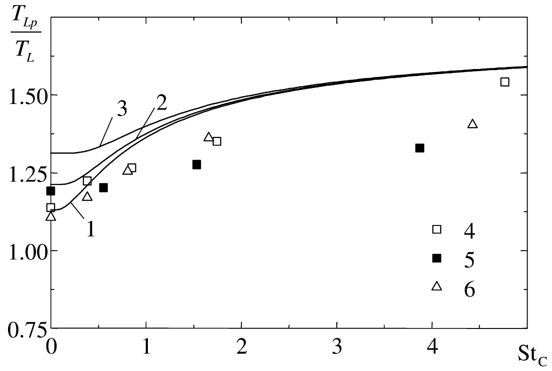

Figure 1.10 Effect of collisions on the duration of particles' interactions with turbulent eddies ($m = 0.5$): 1, 2, 3 – (1.64) with regard to (1.76); 1 – $St_E = 0.25$; 2 – $St_E = 0.5$; 3 – $St_E = 1$; 4–6 – Berlemont *et al.* (1995).

where τ_c is the time interval between collisions. Eq. (1.76) is saying that as the time interval between collisions becomes shorter (in other words, as the collision frequency increases), the effective mean free path of particles decreases as if we increased the particles' inertia. The effect of collisions on the correlations of fluid velocity fluctuations along particle trajectories is determined by the dimensionless parameter $St_C \equiv \tau_p/\tau_c$. This parameter can be considered as the Stokes number characterizing the inertia of particles in relation to the interval between collisions. When the average slip is absent ($\gamma = 0$), the autocorrelation function $\Psi_{Lp}(\tau)$ and the duration of particles' interaction with the turbulence T_{Lp} coincide with the Lagrangian and Eulerian characteristics in the respective limiting cases $St_C \to 0$ and $St_C \to \infty$.

Figure 1.10 illustrates the influence of interparticle collisions on the duration of particles' interactions with turbulent eddies in the absence of the average slip, when T_{Lp} is determined by expression (1.64) with proper account taken of Eq. (1.76). The same figure presents the results of direct stochastic simulation of particle motion in isotropic turbulent field that were obtained by the LES method (Berlemont *et al.*, 1995). The calculations were carried out for particles of diameter $d_p = 656\,\mu m$ and densities $\rho_p = 50$, 100, and $200\,kg/m^3$ moving in air. As it follows from Figure 1.9, instead of being roughly consistent with the LES data, the dependence (1.64) together with Eq. (1.76) predicts stronger effect of collisions on T_{Lp} than suggested by the results of calculations made by Berlemont *et al.* (1995). It is also seen that when St_C gets larger, the effect of inertia governed by parameter St_E takes a smaller role, and the effect of collisions emerges as the predominant factor.

For $m \leq 1$ the integral (1.64) together with Eq. (1.76) can be approximated by an expression similar to Eq. (1.65):

$$T_{Lp} = T_L + (T_E - T_L) f(St_*), \quad f(St_*) = \frac{St_*}{1 + St_*} - \frac{0.9 m St_*^2}{(1 + St_*)^2 (2 + St_*)},$$
$$St_* = \left(St_E^5 + 10\,St_C^5\right)^{1/5}, \tag{1.77}$$

where St_* denotes a parameter characterizing the effect of collisions on the duration of particle–turbulence interaction. The crossing trajectory effect influences T_{Lp}, which can be taken into account by replacing St_E in Eq. (1.71) with the effective Stokes number St_*.

The Lagrangian correlation moment of fluid particle's temperature fluctuations along an inertial particle trajectory is represented in the form similar to Eq. (1.51):

$$\begin{aligned} B_{Ltp}(\tau) &= \langle \vartheta'^2 \rangle \Psi_{Ltp}(\tau) = \langle \vartheta'(\mathbf{x}, t)\vartheta'(\mathbf{R}_p(t-\tau), t-\tau) | \mathbf{R}_p(t) = \mathbf{x} \rangle \\ &= \int \langle \vartheta'(\mathbf{x}, t)\vartheta'(\mathbf{x}-\mathbf{r}, t-\tau)\delta(\mathbf{r}-\mathbf{s}_p(\tau)) \rangle dr, \end{aligned} \tag{1.78}$$

where $\Psi_{Ltp}(\tau)$ is the Lagrangian autocorrelation function of fluid temperature along the particle trajectory.

Corrsin's hypothesis about the possibility of independent statistical averaging of random fields of particle displacements and Euleruan characteristics of the continuum as applied to temperature fluctuations yields

$$\langle \vartheta'(\mathbf{x}, t)\vartheta'(\mathbf{x}-\mathbf{r}, t-\tau)\delta(\mathbf{r}-\mathbf{s}(\tau)) \rangle = \langle \vartheta'(\mathbf{x}, t)\vartheta'(\mathbf{x}-\mathbf{r}, t-\tau) \rangle \phi(\mathbf{r}, \tau). \tag{1.79}$$

Substituting Eq. (1.79) into Eq. (1.78) and employing the relations (1.43), (1.53), and (1.57), we arrive at the following expression for the Lagrangian autocorrelation function of fluid temperature fluctuations along the particle trajectory:

$$\Psi_{Ltp}(\tau) = F_t(s)\Psi_{Et}(\tau), \quad s = \sqrt{W^2\tau^2 + u'^2\psi^2(\tau)}. \tag{1.80}$$

Exponential approximation of Eulerian spatial and temporal correlation functions of temperature (1.44) transforms Eq. (1.80) into

$$\Psi_{Ltp}(\tau) = \exp\left[-\frac{\tau + m_t(\gamma^2\tau^2 + \psi^2(\tau))^{1/2}}{T_{Et}}\right], \tag{1.81}$$

where $m_t \equiv u' T_{Et}/L_t$ is the temperature structure parameter of turbulence.

The integral scale of fluid temperature fluctuations along the inertial particle trajectory is determined through the autocorrelation function (1.81) as

$$T_{Ltp} = \int_0^\infty \Psi_{Ltp}(\tau)\, d\tau = \int_0^\infty \exp\left[-\frac{\tau + m_t(\gamma^2\tau^2 + \psi^2(\tau))^{1/2}}{T_{Et}}\right] d\tau. \tag{1.82}$$

The autocorrelation function (1.81) and time microscale (1.82) take into account both the crossing trajectory effect and the particle inertia effect, which influence temperature fluctuations of the continuum calculated along the particle trajectories. The influence of these two phenomena on velocity fluctuations is governed by the parameters γ and St_E. Collisions between particles can be taken into account in Eq. (1.81) and Eq. (1.82) through the dependence (1.76), whereas the net result of interparticle collisions is characterized by Stokes' parameter St_C. In the limiting case of inertialess particles ($\mathrm{St}_E = \mathrm{St}_C = \gamma = 0$), there follows from Eq. (1.82) a relation between Lagrangian and Eulerian time macroscales of temperature fluctuations in a turbulent fluid:

$$\frac{T_{Lt}}{T_{Et}} = \frac{1}{1+m_t}. \tag{1.83}$$

In accordance with Eq. (1.83) the ratio of scales T_{Lt}/T_{Et} is governed only by the temperature structure parameter of turbulence $m_t \equiv u' T_{Et}/L_t$ similarly to the dependence (1.61) of the ratio of velocity fluctuation scales T_L/T_E on m. A similar dependence of T_{Lt}/T_{Et} on m_t has been obtained by Derevich (2001) on the basis of Corrsin's independence conjecture by using the spectral method.

In the absence of drift, the integral (1.82) can be approximated on the interval $m_t \leq 1$ by the dependence similar to Eq. (1.65),

$$T_{Ltp} = T_{Lt} + (T_{Et} - T_{Lt})\left[\frac{\mathrm{St}_E}{1+\mathrm{St}_E} - \frac{0.8 m_t \mathrm{St}_E^2}{(1+\mathrm{St}_E)^2(2+\mathrm{St}_E)}\right], \tag{1.84}$$

the approximation being asymptotically exact at $m_t \to 0$.

In the limiting cases of very small and large values of the Stokes number St_E, the following asymptotic relations result from Eq. (1.82):

$$T_{Ltp}(\mathrm{St}_E \to 0) = \frac{T_{Et}}{1+m_t(1+\gamma^2)^{1/2}}, \quad T_{Ltp}(\mathrm{St}_E \to \infty) = \frac{T_{Et}}{1+m_t\gamma}. \tag{1.85}$$

In the presence of drift and for the parameter range $m_t \leq 1, 0 \leq \mathrm{St}_E < \infty, 0 \leq \gamma < \infty$, the integral (1.82) can be approximated by a dependence that is based on the relations (1.84) and (1.85), with the margin of error not exceeded 5% and the end result being similar to Eq. (1.71):

$$\begin{aligned} T_{Ltp} &= T_{Ltp}(\mathrm{St}_E \to 0) + [T_{Ltp}(\mathrm{St}_E \to \infty) - T_{Ltp}(\mathrm{St}_E \to 0)] \\ &\times \left[\frac{\mathrm{St}_E}{1+\mathrm{St}_E} - \frac{0.8 m_t \mathrm{St}_E^2}{(1+\mathrm{St}_E)^2(2+\mathrm{St}_E)}\right]. \end{aligned} \tag{1.86}$$

The effect of interparticle collisions on T_{Ltp} can be taken into account by replacing St_E in Eq. (1.84) and Eq. (1.86) with the effective Stokes number St_*.

Similarly to Eq. (1.67), the function Ψ_{Ltp} of τ/T_{Ltp} is rather accurately described by the exponential dependence

$$\Psi_{Ltp}(\tau) = \exp\left(-\frac{\tau}{T_{Ltp}}\right) \tag{1.87}$$

which is asymptotically exact in the limit $\mathrm{St}_* \to \infty$ and $\gamma \to \infty$.

By making an appropriate preliminary choice of the autocorrelation functions, say, in the form given by the exponential functions (1.67), (1.75), and (1.87), we can reduce the problem of finding fluid velocity and temperature correlations along inertial particle trajectories to the problem of finding the Lagrangian time scales $\mathbf{T}_{Lp}$ and T_{Ltp} characterizing the interaction of particles with turbulent eddies.

It should be noted that the model proposed in the present section requires estimations for $\mathbf{T}_{Lp}$ and T_{Ltp} and has a semi-empirical character: not only does it hinge on the previously made assumptions, in particular, on Corrsin's conjecture employed in Eq. (1.52), Eq. (1.79) and especially on the suppositions we made when deriving Eq. (1.57) and Eq. (1.76), but it also critically depends on the structure parameters of turbulence m and m_t that must be provided externally, that is, obtained from a physical experiment or from numerical simulation.

1.4 Velocity and Temperature Correlations for Particles in Stationary Isotropic Turbulence

The present section considers the correlation moments for the velocity and temperature of particles in a stationary isotropic turbulent field and presents the relations between the intensities of velocity and temperature fluctuations in the disperse and continuous phases.

First, let us define the mixed Lagrangian correlation moment of fluid and particle velocity fluctuations:

$$B_{fp\,ij}(\tau) = \langle u_i'(\mathbf{x},t)v_j'(\mathbf{R}_p(t-\tau),t-\tau)|\mathbf{R}_p(t)=\mathbf{x}\rangle, \tag{1.88}$$

where $\mathbf{R}_p$ is the position vector of a point on the particle's trajectory.

From the particle's equation of motion (1.46) there follows an equation for the mixed correlation moment (1.88):

$$\frac{dB_{fp\,ij}}{d\tau} - \frac{B_{fp\,ij}}{\tau_p} = -\frac{B_{Lp\,ij}}{\tau_p}, \tag{1.89}$$

where $B_{Lp\,ij}\,(\tau)$ is the Lagrangian correlation moment of fluid velocity fluctuations along the particle's trajectory (1.51). The solution of Eq. (1.89) obeying the condition $B_{fp\,ij} \to 0$ at $\tau \to \infty$ is written in the matrix notation as

$$\mathbf{B}_{fp}(\tau) = \frac{1}{\tau_p}\int_\tau^\infty \mathbf{B}_{Lp}(\xi)\exp\left(\frac{\tau-\xi}{\tau_p}\mathbf{I}\right)d\xi = \frac{u'^2}{\tau_p}\int_\tau^\infty \Psi_{Lp}(\xi)\exp\left(\frac{\tau-\xi}{\tau_p}\mathbf{I}\right)d\xi, \tag{1.90}$$

where $\mathbf{I}$ is the unit matrix.

The relation (1.90) gives an expression for the mixed single-point moment of velocity fluctuations of the continuous and disperse phases:

$$\langle u_i'v_j'\rangle = B_{fp\,ij}(0) = u'^2 f_{u\,ij}, \tag{1.91}$$

where

$$\mathbf{f}_u = \frac{1}{\tau_p}\int_0^\infty \Psi_{Lp}(\tau)\exp\left(-\frac{\tau}{\tau_p}\mathbf{I}\right)d\tau. \tag{1.92}$$

The Lagrangian correlation moment of velocity fluctuations for a particle along its trajectory looks as follows:

$$B_{p\,ij}(\tau) = \langle v_i'(\mathbf{x},t)v_j'(\mathbf{R}_p(t-\tau),t-\tau)|\mathbf{R}_p(t)=\mathbf{x}\rangle. \tag{1.93}$$

In view of the relation

$$\left\langle \frac{dv'_i(t)}{dt}\frac{dv'_j(t-\tau)}{dt}\right\rangle = -\frac{d^2B_{p\,ij}}{d\tau^2}$$

the particle's equation of motion (1.46) leads to the following equation for the correlation moment of particle velocity fluctuations:

$$\frac{d^2B_{p\,ij}}{d\tau^2} - \frac{B_{p\,ij}}{\tau_p^2} = -\frac{B_{Lp\,ij}}{\tau_p^2}. \tag{1.94}$$

The following boundary conditions apply for Eq. (1.94):

$$\frac{dB_{p\,ij}}{d\tau} = 0 \quad \text{at} \quad \tau = 0, \quad B_{p\,ij} \to 0 \quad \text{at} \quad \tau \to \infty. \tag{1.95}$$

Integrating Eq. (1.94) and making use of the boundary conditions (1.95), we get an asymptotic expression for the tensor of turbulent diffusion of particles for large values of time:

$$D_{p\,ij} = \int_0^\infty B_{p\,ij}(\tau)d\tau = \int_0^\infty B_{Lp\,ij}(\tau)d\tau = u'^2T_{Lp\,ij}. \tag{1.96}$$

In accordance with Eq. (1.96), the tensor of turbulent diffusion of inertial particles coincides with the corresponding quantity for fluid particles moving along inertial particle trajectories. Comparing Eq. (1.2) and Eq. (1.96), we obtain

$$D_{p\,ij} = \frac{T_{Lp\,ij}}{T_L}D_t. \tag{1.97}$$

Thus the ratio of turbulent diffusion coefficients for fluid and solid (inertial) particles is equal to the ratio of integral scales of fluctuation velocities of the continuous carrier medium calculated along the corresponding particle trajectories. If we ignore the distinction between Lagrangian integral scales of turbulence along the trajectories of fluid and solid particles, in other words, if we assume $T_{Lp\,ij} = T_L\delta_{ij}$, then, as it follows from Eq. (1.97), the turbulent diffusion tensors for fluid and solid particles will also coincide. This result was first obtained by Chen (Hinze, 1975). From Eq. (1.97) it follows that in the absence of the average slip (drift) between the particles and the fluid, when the duration of particle interaction with turbulent eddies T_{Lp} exceeds the Lagrangian macroscale T_L, the coefficient of turbulent diffusion turns out to be greater for solid particles than for fluid particles. This phenomenon is called the inertia effect (Reeks, 1977; Pismen and Nir, 1978; Deutsch and Simonin, 1991; Squires and Eaton, 1991b; Elghobashi and Truesdell, 1992). With increase of the average slip, the duration of particle interaction with turbulent eddies decreases and thus the turbulent diffusion coefficient decreases as well. This effect is referred to as the crossing trajectory effect (Yudine, 1959; Csanady, 1963). Besides, the time scale T^l_{Lp} has turned out to be greater than T^n_{Lp}, therefore the diffusion coefficient of particles in the longitudinal direction (with respect to the drift velocity vector) D^l_p

exceeds the diffusion coefficient in the transverse direction D_p^n. This effect is called the continuity effect (Csanady, 1963).

Taking Eq. (1.95) into account, we find the solution of Eq. (1.94):

$$\mathbf{B}_p(\tau) = \frac{u'^2}{2\tau_p}\int_0^\infty \left[\exp\left(-\frac{|\tau+\xi|}{\tau_p}\mathbf{I}\right) + \exp\left(-\frac{|\tau-\xi|}{\tau_p}\mathbf{I}\right)\right]\mathbf{\Psi}_{Lp}(\xi)\, d\xi. \tag{1.98}$$

The expression (1.98) was first obtained by Reeks (1977) by direct integration of equations of motion for the particles and invoking Corrsin's hypothesis about independent averaging of particle displacement fields and fluid velocity fluctuations. In accordance with Eq. (1.98), a single-point second moment of particle fluctuation velocities (turbulent stress tensor) is given by the expression similar to Eq. (1.91):

$$\langle v_i' v_j' \rangle = B_{p\,ij}(0) = u'^2 f_{u\,ij}. \tag{1.99}$$

The tensor $f_{u\ ij}$ in Eq. (1.91) and Eq. (1.99) characterizes the extent of particles' involvement in fluctuational motion of the turbulent carrier medium. Thus inertia-less particles ($\tau_p \to 0$) are completely involved in turbulent motion and their kinetic energy coincides with the turbulent energy of the carrier fluid. If the particles are inertialess, then Eq. (1.92) gives us

$$\lim_{\tau_p \to 0} f_{u\,ij} = \delta_{ij},$$

since $\Psi_{Lp\,ij}(0) = \delta_{ij}$.

The extent of particles' involvement in fluctuational motion decreases as their inertia gets higher, and thus kinetic energy of inertial particles in homogeneous isotropic stationary turbulence is always lower than turbulent energy of the fluid. For highly inertial particles, if follows from Eq. (1.92) that

$$\lim_{\tau_p \to \infty} f_{u\,ij} = \frac{T_{Lp\,ij}}{\tau_p}.$$

If the Lagrangian autocorrelation function of fluid velocity along the particle's path $\Psi_{Lp\,ij}(\tau)$is described by the exponential approximation (1.75), the correlation moment of particle velocity fluctuations (1.98) takes the form

$$\mathbf{B}_p(\tau) = \frac{u'^2}{2}\left\{(\mathbf{I}+\tau_p\mathbf{T}_{Lp}^{-1})^{-1}\left[\exp(-\tau\mathbf{T}_{Lp}^{-1}) + \exp\left(-\frac{\tau}{\tau_p}\mathbf{I}\right)\right]\right.$$
$$\left. + (\mathbf{I}-\tau_p\mathbf{T}_{Lp}^{-1})^{-1}\left[\exp(-\tau\mathbf{T}_{Lp}^{-1}) - \exp\left(-\frac{\tau}{\tau_p}\mathbf{I}\right)\right]\right\}, \tag{1.100}$$

and the tensor characterizing the extent of particles' involvement in turbulent motion (1.92) becomes equal to

$$\mathbf{f}_u = (\mathbf{I}+\tau_p\mathbf{T}_{Lp}^{-1})^{-1}. \tag{1.101}$$

In the absence of the average slip of particles with respect to the turbulent fluid, the Lagrangian correlation moments (1.88), (1.93) and the tensor of particles' involvement in turbulent motion (1.92) turn out to be isotropic:

$$B_{fp\,ij}(\tau) = B_{fp}(\tau)\delta_{ij},\ B_{p\,ij}(\tau) = B_p(\tau)\delta_{ij},\ f_{u\,ij} = f_u\delta_{ij},$$
$$f_u = \frac{1}{\tau_p}\int_0^\infty \Psi_{Lp}(\tau)\exp\left(-\frac{\tau}{\tau_p}\right)d\tau. \tag{1.102}$$

It is evident that in this case the single-point moment of velocity fluctuations of the continuous and disperse phases (1.91) and the single-point moment of velocity fluctuations of particles (1.99) will be isotropic as well:

$$\langle u'_i v'_j\rangle = \langle v'_i v'_j\rangle = v'^2\delta_{ij}, \tag{1.103}$$

where $v'^2 \equiv \langle v'_n v'_n\rangle/3$ is the intensity of particle velocity fluctuations defined as

$$v'^2 = f_u u'^2. \tag{1.104}$$

In the case of an exponential autocorrelation function the isotropic involvement coefficient (1.102) simplifies to

$$f_u = \left(1 + \frac{\tau_p}{T_{Lp}}\right)^{-1}. \tag{1.105}$$

The relation (1.104) for the intensity ratio of velocity fluctuations of the disperse and continuous phases with the involvement factor given by (1.105) at $T_{Lp} = T_L$ was first obtained by Chen (Hinze, 1975). As evidenced by Figure 1.11, the dependence (1.104) with the involvement coefficient given by (1.105) is in very good agreement with the results of numerical simulation by the LES method (Deutsch and Simonin, 1991).

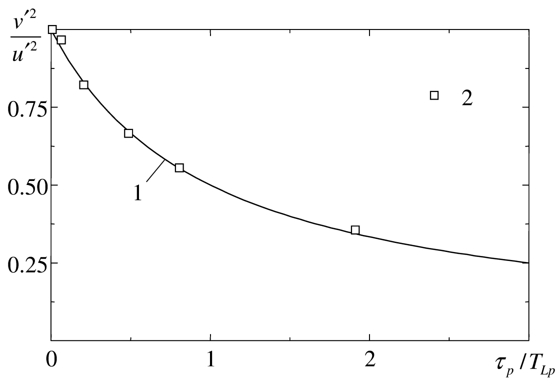

Figure 1.11 Relation between the intensities of velocity fluctuations of the disperse and continuous phases: 1 – (1.104) and (1.105); 2 – Deutsch and Simonin (1991).

Define the Lagrangian autocorrelation function of particle velocities in the absence of the average drift as $\Psi_p(\tau) = B_p(\tau)/B_p(0)$. Then, by virtue of Eq. (1.98) and Eq. (1.102) the Lagrangian integral scale for particles is

$$T_p = \int_0^\infty \Psi_p(\tau)d\tau = \frac{T_{Lp}}{f_u}. \tag{1.106}$$

If we set the autocorrelation function of fluid particles moving along the inertial particle trajectories equal to the exponential approximation of the autocorrelation function of inertial particle velocities, then Eq. (1.100) gives

$$\begin{aligned}\Psi_p(\tau) &= \frac{1}{2}\left[\exp\left(-\frac{\tau}{T_{Lp}}\right) + \exp\left(-\frac{\tau}{\tau_p}\right)\right] \\ &+ \frac{(T_{Lp}+\tau_p)}{2(T_{Lp}-\tau_p)}\left[\exp\left(-\frac{\tau}{T_{Lp}}\right) - \exp\left(-\frac{\tau}{\tau_p}\right)\right],\end{aligned} \tag{1.107}$$

and the Lagrangian integral scale (1.106) becomes equal to

$$T_p = T_{Lp} + \tau_p. \tag{1.108}$$

Figure 1.12 compares the formula (1.107) with the DNS results at $\mathrm{Re}_\lambda = 53$ (Simonin *et al.*, 2002). The duration of particles' interaction with turbulent eddies entering Eq. (1.107) was determined from Eq. (1.65). The DNS results suggest $T_L/T_E = 0.68$, hence the theoretical curves corresponding to Eq. (1.107) correspond to the value of the structure parameter $m = 0.3$, which due to Eq. (1.61) gives a value that is close to the above-mentioned ratio of the Lagrangian and Eulerian turbulence scales. The main conclusion following from Figure 1.12 is that the Lagrangian integral time scale of particle velocity fluctuations grows with particle inertia, which is in good agreement with the formulas (1.106) and (1.108). For highly inertial particles, the Lagrangian macroscale T_p becomes equal to the relaxation time τ_p.

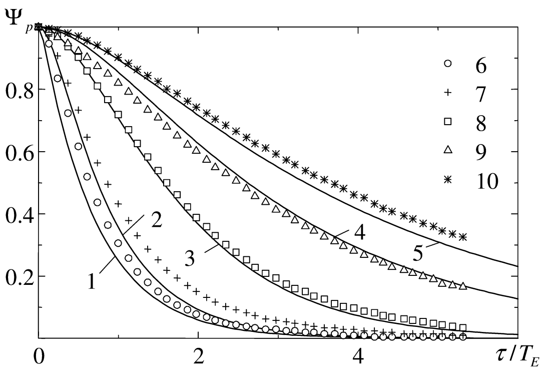

Figure 1.12 Autocorrelation function of particle velocity fluctuations: 1–5 – (1.107); 6–10 – Simonin *et al.* (2002); 1, 6 – $\mathrm{St}_E = 0.04$; 2, 7 – $\mathrm{St}_E = 0.2$; 3, 8 – $\mathrm{St}_E = 1.0$; 4, 9 – $\mathrm{St}_E = 2.3$; 5, 10 – $\mathrm{St}_E = 3.3$.

We now turn to the discussion of temperature fluctuations. The Lagrangian mixed correlation moment of temperatures of the continuous and disperse phases along the particle trajectories is defined as

$$B_{fpt}(\tau) = \langle \vartheta'_i(\mathbf{x}, t)\theta'_j(\mathbf{R}_p(t-\tau), t-\tau)|\mathbf{R}_p(t) = \mathbf{x}\rangle. \tag{1.109}$$

The equation for B_{fpt} is derived directly from the heat exchange equation (1.49) for a single particle:

$$\frac{dB_{fpt}}{d\tau} - \frac{B_{fpt}}{\tau_t} = -\frac{B_{Ltp}}{\tau_t}, \tag{1.110}$$

where B_{Ltp} is the Lagrangian correlation moment of temperature fluctuations of a fluid particle moving along the inertial particle trajectory (see Eq. (1.77)). The solution of Eq. (1.110) has the form similar to Eq. (1.90):

$$B_{fpt}(\tau) = \frac{1}{\tau_t}\int_\tau^\infty B_{Ltp}(\xi)\exp\left(\frac{\tau-\xi}{\tau_t}\right)d\xi = \frac{\langle \vartheta'^2\rangle}{\tau_t}\int_\tau^\infty \Psi_{Ltp}(\xi)\exp\left(\frac{\tau-\xi}{\tau_t}\right)d\xi. \tag{1.111}$$

According to Eq. (1.111), the mixed single-point moment of temperature fluctuations of the continuous and disperse phases is

$$\langle \vartheta'\theta'\rangle = B_{fpt}(0) = f_t\langle \vartheta'^2\rangle, \tag{1.112}$$

where

$$f_t = \frac{1}{\tau_t}\int_0^\infty \Psi_{Ltp}(\tau)\exp\left(-\frac{\tau}{\tau_t}\right)d\tau. \tag{1.113}$$

The Lagrangian correlation moment of temperature fluctuations of a particle along its trajectory is equal to

$$B_{pt}(\tau) = \langle \theta'(\mathbf{x}, t)\theta'(\mathbf{R}_p(t-\tau), t-\tau)|\mathbf{R}_p(t) = \mathbf{x}\rangle. \tag{1.114}$$

In view of the fact that

$$\left\langle \frac{d\theta'(t)}{dt}\frac{d\theta'(t-\tau)}{dt}\right\rangle = -\frac{d^2B_{pt}}{d\tau^2},$$

Equation (1.49) yields the following equation for the correlation moment of particle temperature fluctuations:

$$\frac{d^2B_{pt}}{d\tau^2} - \frac{B_{pt}}{\tau_t^2} = -\frac{B_{Ltp}}{\tau_t^2} \tag{1.115}$$

whose solution has the form similar to Eq. (1.98),

$$B_{pt}(\tau) = \frac{\langle \vartheta'^2\rangle}{2\tau_t}\int_0^\infty \left[\exp\left(-\frac{|\tau+\xi|}{\tau_t}\right) + \exp\left(-\frac{|\tau-\xi|}{\tau_t}\right)\right]\Psi_{Ltp}(\xi)d\xi. \tag{1.116}$$

Due to Eq. (1.116), second single-point moments of temperature fluctuations of the disperse and continuous phases are connected by the relation

$$\langle \theta'^2 \rangle = B_{pt}(0) = f_t \langle \vartheta'^2 \rangle, \tag{1.117}$$

where the coefficient f_t (see Eq. (1.113)) characterizes the susceptibility of particles to turbulent fluctuations of the carrier fluid temperature.

If the Lagrangian autocorrelation function of fluid temperature along the particle trajectory $\Psi_{Ltp}(\tau)$ is described by the exponential approximation (1.87), the correlation moment of the particle's temperature fluctuations (1.116) becomes

$$B_{pt}(\tau) = \frac{\langle \vartheta'^2 \rangle}{2} \left\{ \left(1 + \frac{\tau_t}{T_{Ltp}}\right)^{-1} \left[\exp\left(-\frac{\tau}{T_{Ltp}}\right) + \exp\left(-\frac{\tau}{\tau_t}\right) \right] + \left(1 - \frac{\tau_t}{T_{Ltp}}\right)^{-1} \left[\exp\left(-\frac{\tau}{T_{Ltp}}\right) - \exp\left(-\frac{\tau}{\tau_t}\right) \right] \right\},$$

and the coefficient of particles' involvement in temperature fluctuations of the continuum (1.113) reduces to

$$f_t = \left(1 + \frac{\tau_t}{T_{Ltp}}\right)^{-1}. \tag{1.118}$$

After some algebra, the formulas (1.104), (1.105), (1.117), and (1.118) give us the following relation between turbulent fluctuations of temperature and velocities of the disperse and continuous phases:

$$\frac{\langle \theta'^2 \rangle}{\langle \vartheta'^2 \rangle} = \frac{v'^2/u'^2}{v'^2/u'^2 + \Upsilon(1 - v'^2/u'^2)}, \quad \Upsilon = \frac{\tau_t T_{Lp}}{\tau_p T_{Ltp}}, \tag{1.119}$$

where the parameter Υ characterizes the relation between heat and the dynamic inertia of the particles.

By performing a simple comparison of solutions of the equations for the velocity and particle temperature fluctuations, Yarin and Hetsroni (1994) have obtained another relation between turbulent fluctuations of temperature and velocities of the disperse and continuous phases:

$$\left(\frac{\langle \theta'^2 \rangle}{\langle \vartheta'^2 \rangle}\right)^{1/2} = 1 - \left(1 - \frac{v'}{u'}\right)^{\frac{\tau_p}{\tau_t}}. \tag{1.120}$$

Starting from Eq. (1.104) and Eq. (1.117) and employing step autocorrelation functions to determine the involvement coefficients (1.102) and (1.113), Derevich (2001) also obtained a relation between turbulent fluctuations of temperature and velocities of the disperse and continuous phases:

$$\frac{\langle \theta'^2 \rangle}{\langle \vartheta'^2 \rangle} = 1 - \left(1 - \frac{v'^2}{u'^2}\right)^{\frac{1}{\Upsilon}}, \tag{1.121}$$

which is identical in form to the relation (1.120) but was derived more rigorously.

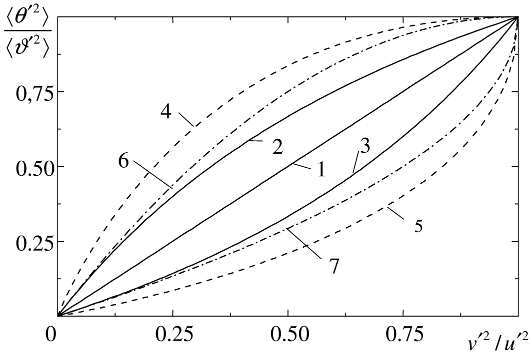

Figure 1.13 Ratio of temperature fluctuation intensities of the disperse and continuous phases vs the ratio of velocity fluctuation intensities: 1 – $\Upsilon = 1$; 2, 4, 6 – $\Upsilon = 0.5$; 3, 5, 7 – $\Upsilon = 2$; 2, 3 – (1.119); 4, 5 – (1.120); 6, 7 – (1.121).

Figure 1.13 presents the ratio of temperature fluctuation intensities of the disperse and continuous phases as a function of the parameter Υ. It can be seen that the behavior of the dependences (1.119), (1.120), and (1.121) is qualitatively identical in the sense that all of them predict that $\langle \theta'^2 \rangle / \langle \vartheta'^2 \rangle < v'^2/u'^2$ at $\Upsilon < 1$ and that $\langle \theta'^2 \rangle / \langle \vartheta'^2 \rangle > v'^2/u'^2$ at $\Upsilon > 1$. Figure 1.14 shows how the parameter Υ, which characterizes the ratio between heat inertia and dynamic inertia of the particle, influences the ratio of temperature fluctuation intensities of the disperse and continuous phases when the ratio of velocity fluctuation intensities remains fixed ($v'^2/u'^2 = 0.71$). It stands out that Eq. (1.119) is in very good agreement with the DNS results (Jaberi, 1998) whereas the dependence (1.120), when compared to the DNS data, predicts an excessively strong decrease of $\langle \theta'^2 \rangle / \langle \vartheta'^2 \rangle$ with increase of Υ – a fact that has been already noted by Jaberi (1998).

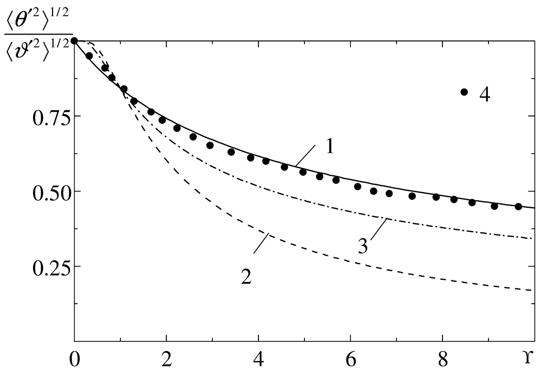

Figure 1.14 Ratio of temperature fluctuation intensities of the disperse and continuous phases vs the ratio between heat and the dynamic inertia of the particles: 1 – (1.119), 2 – (1.120), 3 – (1.121), 4 – Jaberi (1998).

1.5
Particle Acceleration in Isotropic Turbulence

In the present section we discuss the statistics of low-inertia particle acceleration under the assumption that the deviation of particle trajectories from those of fluid (inertialess) particles can be neglected. Let us first define the Lagrangian correlation moment of fluid particle acceleration fluctuations,

$$A_{L\,ij}(\tau) = \left\langle \frac{du'_i(\mathbf{x},t)}{dt}\frac{du'_j(\mathbf{R}(t-\tau),t-\tau)}{dt}|\mathbf{R}(t)=\mathbf{x}\right\rangle = \langle a'_i a'_j\rangle \Psi_a(\tau), \tag{1.122}$$

where $\langle a'_i a'_j\rangle$ is the variation of acceleration fluctuations, and $\Psi_a(\tau)$ is the autocorrelation function of acceleration.

As a consequence of the kinematic relation $A_{L\,ij} = -d^2 B_{L\,ij}/d\tau^2$ and Eq. (1.1), the correlation of acceleration fluctuations (1.122) in isotropic turbulence manifests itself as

$$A_{L\,ij}(\tau) = -\Psi''_L(\tau)u'^2\delta_{ij}. \tag{1.123}$$

Taking into account the normalization condition $\Psi_a(0)=1$, we derive from Eq. (1.123) the expressions for the variance and the autocorrelation function of acceleration fluctuations:

$$\langle a'_i a'_j\rangle = \frac{2u'^2\delta_{ij}}{\tau_T^2} = \frac{a_0\varepsilon^{3/2}\delta_{ij}}{\nu^{1/2}}, \quad \Psi_a(\tau) = \frac{\Psi''_L(\tau)}{\Psi''_L(0)} = -\frac{\tau_T^2\Psi''_L(\tau)}{2}, \tag{1.124}$$

where τ_T is the Taylor differential time scale (1.6).

The Lagrangian correlation moment of acceleration fluctuations for inertial particles is defined as

$$A_{p\,ij}(\tau) = \left\langle \frac{dv'_i(\mathbf{x},t)}{dt}\frac{dv'_j(\mathbf{R}_p(t-\tau),t-\tau)}{dt}|\mathbf{R}_p(t)=\mathbf{x}\right\rangle = \langle a'_{pi} a'_{pj}\rangle \Psi_{pa}(\tau). \tag{1.125}$$

We conclude from the self-evident relation $A_{p\,ij} = -d^2 B_{p\,ij}/d\tau^2$ and from Eq. (1.47) that the correlation of the particles' acceleration fluctuations (1.125) is equal to

$$A_{p\,ij}(\tau) = \frac{B_{Lp\,ij}(\tau)-B_{p\,ij}(\tau)}{\tau_p^2} = \frac{[\Psi_{Lp}(\tau)u'^2 - \Psi_p(\tau)v'^2]\delta_{ij}}{\tau_p^2} = \langle a'_{pi}a'_{pj}\rangle\Psi_{pa}(\tau). \tag{1.126}$$

Expression (1.126) together with the condition $\Psi_{pa}(0)=1$ gives us the variance of the fluctuation as well as the autocorrelation function for the acceleration of inertial particles:

$$\langle a'_{pi}a'_{pj}\rangle = \frac{(u'^2-v'^2)\delta_{ij}}{\tau_p^2} = \frac{(1-f_u)u'^2\delta_{ij}}{\tau_p^2}, \quad \Psi_{pa}(\tau) = \frac{\Psi_{Lp}(\tau)-\Psi_p(\tau)f_u}{1-f_u}. \tag{1.127}$$

By analogy with the first relation (1.124), let us represent the fluctuation variance of inertial particles' acceleration as

$$\langle a'_{pi}a'_{pj}\rangle = \frac{a_{p0}\varepsilon^{3/2}\delta_{ij}}{\nu^{1/2}}, \tag{1.128}$$

where a_{p0} is the dimensionless amplitude of acceleration fluctuations. Comparing the first relations in Eq. (1.127) and Eq. (1.128), we get

$$a_{p0} = \frac{(1-f_u)\mathrm{Re}_\lambda}{15^{1/2}\mathrm{St}^2}, \tag{1.129}$$

where $\mathrm{St} \equiv \tau_p/\tau_k$ is the Stokes number determined by the Kolmogorov time microscale.

The coefficient of particles' involvement in turbulent motion entering Eq. (1.129) is defined by Eq. (1.102). The autocorrelation function $\Psi_{Lp}(\tau)$ is taken to be equal to $\Psi_L(\tau)$, which in its turn is given by the bi-exponential approximation (1.5). Accordingly, the involvement coefficient takes the form

$$f_u = \frac{2\mathrm{St}_L + z^2}{2\mathrm{St}_L + 2\mathrm{St}_L^2 + z^2}, \tag{1.130}$$

where $\mathrm{St}_L \equiv \tau_p/T_L$ is the Stokes number determined by the Lagrangian time macroscale. As the Reynolds number gets larger, $z \to 0$ and the involvement coefficient tends to

$$f_u = \frac{1}{1+\mathrm{St}_L}, \tag{1.131}$$

which corresponds to the exponential autocorrelation function (1.3).

Substituting Eq. (1.130) into Eq. (1.129) and making use of Eq. (1.6), we get

$$a_{p0} = a_0\left[1 + \frac{15^{1/2}a_0\mathrm{St}(\mathrm{St}+T_L)}{\mathrm{Re}_\lambda}\right]^{-1}. \tag{1.132}$$

Figure 1.15, which plots the dimensionless amplitude of particle acceleration against the Stokes number, compares the dependence predicted by formula (1.132)

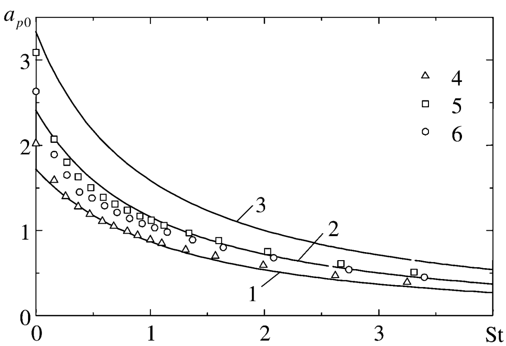

Figure 1.15 Amplitude of particle acceleration fluctuations: 1, 2, 3 – (1.132); 4, 5, 6 – Bec *et al.* (2006); 1, 4 – $\mathrm{Re}_\lambda = 65$; 2, 5 – $\mathrm{Re}_\lambda = 105$; 3, 6 – $\mathrm{Re}_\lambda = 185$.

(with Eqs. (1.4) and (1.7)) taken into consideration) with the DNS results obtained by Bec *et al.* (2006). It is readily seen that Eq. (1.132) describes the influence of St as well as Re_λ with a sufficient accuracy. It is obvious that since particle acceleration is governed primarily by small-scale turbulent structures, it makes sense to describe a particle's inertia in terms of the relaxation time divided by the Kolmogorov microscale – in contrast to the effect of particle inertia on the intensity of velocity fluctuations, which is better described in terms of the time macroscale of turbulence. As a consequence, we cannot avoid mentioning the fact at low values of St, insertion of the involvement coefficient (1.131) into Eq. (1.129) rather than into Eq. (1.130) will produce results that are unacceptable even in the qualitative sense.