

## Contents

**Preface to the Second Edition** XV

**Preface to the First Edition** XVII

### **Part I** Problems 1

#### **1** Crystal Structures 3

- 1.1 Problem: Symmetries 8
- 1.2 Problem: Rotations 8
- 1.3 Problem: Copper Oxide Layers 9
- 1.4 Problem: Graphite 10
- 1.5 Problem: Structure of  $A_xC_{60}$  10
- 1.6 Problem: *hcp* and *fcc* Structures 10
- 1.7 Problem: *hcp* and *bcc* Structures 11
- 1.8 Problem: Structure Factor of  $A_xC_{60}$  11
- 1.9 Problem: Neutron Diffraction Device 11
- 1.10 Problem: Linear Array of Emitters: Finite Size Effects 11
- 1.11 Problem: Linear Array of Emitters: Superlattice 12
- 1.12 Problem: Powder Diffraction of *hcp* and *fcc* Crystals 12
- 1.13 Problem: Momentum Resolution 12
- 1.14 Problem: Finite Size Effects 12
- 1.15 Problem: Random Displacement 13
- 1.16 Problem: Vacancies 13
- 1.17 Problem: Integrated Scattering Intensity 13

#### **2** Interatomic Forces and Lattice Vibrations 15

- 2.1 Problem: Madelung Constant 19
- 2.2 Problem: NaCl Bulk Modulus 19
- 2.3 Problem: Madelung with Screened Potential 19
- 2.4 Problem: Triple-axis Spectrometer 19
- 2.5 Problem: Phonons in Silicon 20
- 2.6 Problem: Linear Array of Emitters: Phonons 20

VI | Contents

2.7	Problem: Long-range Interaction	21
2.8	Problem: Mass Defect	21
2.9	Problem: Debye Frequency	21
2.10	Problem: Vibrations of a Square Lattice	21
2.11	Problem: Grüneisen Parameter	22
2.12	Problem: Diatomic Chain	22
2.13	Problem: Damped Oscillation	23
2.14	Problem: Two-dimensional Debye	23
2.15	Problem: Soft Optical Phonons	23
2.16	Problem: Soft Phonons Again	23
2.17	Problem: Nanowire Phonons	24
<b>3</b>	<b>Electronic Band Structure</b>	<b>25</b>
3.1	Problem: Nearly Free Electrons in One Dimension	30
3.2	Problem: Eigenfunctions of Nearly Free Electrons in One Dimension	30
3.3	Problem: Tight-binding Hamiltonian	31
3.4	Problem: Nearly Free Electrons in Dirac-delta Potentials	31
3.5	Problem: Tight-binding in Dirac-delta Potentials	31
3.6	Problem: Dirac-delta Potentials	32
3.7	Problem: Band Overlap	32
3.8	Problem: Nearly Free Electrons in Two Dimensions	32
3.9	Problem: Nearly Free-electron Bands	33
3.10	Problem: Instability at the Fermi Wavenumber	33
3.11	Problem: Electrons in 2D Nearly Free-electron Band	33
3.12	Problem: Square Lattice	34
3.13	Problem: Tight-binding Band in Two Dimensions	34
3.14	Problem: Electrons in 2D Tight-binding Band	34
3.15	Problem: Dirac-delta Potentials in Two Dimensions	34
3.16	Problem: Graphene	35
3.17	Problem: Effective Mass	35
3.18	Problem: Cyclotron Frequency	35
3.19	Problem: de Haas–van Alphen	35
3.20	Problem: Fermi Energy	36
<b>4</b>	<b>Density of States</b>	<b>37</b>
4.1	Problem: Density of States	39
4.2	Problem: Two-dimensional Density of States	40
4.3	Problem: Two-dimensional Tight Binding	40
4.4	Problem: Tight Binding from One to Three Dimensions	40
4.5	Problem: Crossover to Quasi-one-dimensional Metal	41
4.6	Problem: Phonon Mode of Two-dimensional System	42
4.7	Problem: Saddle Point	42
4.8	Problem: Density of States in Superconductors	43
4.9	Problem: Energy Gap	43

4.10	Problem: Density of States for Hybridized Bands	43
4.11	Problem: Infinite-dimensional DOS	44
4.12	Problem: Two- to Three-dimensional Crossover	44
4.13	Problem: One- to Two-dimensional Crossover	45
4.14	Problem: Carbon Nanotube	45
<b>5</b>	<b>Elementary Excitations</b>	<b>47</b>
5.1	Problem: Tight-binding Model	50
5.2	Problem: Impurity State in the One-dimensional Tight-binding Model	51
5.3	Problem: Hybridization of Energy Bands	51
5.4	Problem: Polarons	52
5.5	Problem: Polaritons	52
5.6	Problem: Excitons	52
5.7	Problem: Holstein–Primakoff Transformation	53
5.8	Problem: Dyson–Maleev Representation	53
5.9	Problem: Spin Waves	53
5.10	Problem: Spin Waves Again	54
5.11	Problem: Anisotropic Heisenberg Model	54
5.12	Problem: Solitons	54
5.13	Problem: Dirac Fermions in Graphene	56
<b>6</b>	<b>Thermodynamics of Noninteracting Quasiparticles</b>	<b>57</b>
6.1	Problem: Specific Heat of Metals and Insulators	64
6.2	Problem: Number of Phonons	64
6.3	Problem: Energy of the Phonon Gas	64
6.4	Problem: Bulk Modulus of the Phonon Gas	64
6.5	Problem: Phonons in One Dimension	64
6.6	Problem: Electron–Hole Symmetry	65
6.7	Problem: Entropy of the Noninteracting Electron Gas	65
6.8	Problem: Free Energy with Gap at the Fermi Energy	65
6.9	Problem: Bulk Modulus at $T = 0$	66
6.10	Problem: Temperature Dependence of the Bulk Modulus	66
6.11	Problem: Chemical Potential of the Free-electron Gas	66
6.12	Problem: EuO Specific Heat	66
6.13	Problem: Magnetization at Low Temperatures	66
6.14	Problem: Electronic Specific Heat	66
6.15	Problem: Quantum Hall Effect	67
<b>7</b>	<b>Transport Properties</b>	<b>69</b>
7.1	Problem: Temperature-dependent Resistance	76
7.2	Problem: Conductivity Tensor	76
7.3	Problem: Montgomery Method	76
7.4	Problem: Anisotropic Layer	77
7.5	Problem: Two-charge-carrier Drude Model	78

VIII | Contents

- 7.6 Problem: Thermal Conductivity 78
- 7.7 Problem: Residual Resistivity 78
- 7.8 Problem: Electric and Heat Transport 79
- 7.9 Problem: Conductivity of Tight-binding Band 79
- 7.10 Problem: Hall Effect in Two-dimensional Metals 80
- 7.11 Problem: Free-electron Results from the Boltzmann Equations 80
- 7.12 Problem:  $p$ - $n$  Junctions 80
- 7.13 Problem: Mott–Ioffe–Regel 81
- 8 Optical Properties 83**
- 8.1 Problem: Reflectivity and Transmission: Multiple Interfaces 93
- 8.2 Problem: Group Velocity 93
- 8.3 Problem: Tinkham Formula 94
- 8.4 Problem: Transmission of a Thin Superconductor 94
- 8.5 Problem: Fourier Transform Infrared Spectroscopy 95
- 8.6 Problem: Sum Rule: Dielectric Constant 96
- 8.7 Problem: Optical Mode of KBr 96
- 8.8 Problem: Direct-gap Semiconductor 96
- 8.9 Problem: Inversion Symmetry 97
- 8.10 Problem: Frequency-dependent Conductivity 97
- 8.11 Problem: Frequency-dependent Response of a Superconductor 97
- 8.12 Problem: Optical Conductivity Tensor 98
- 8.13 Problem: Joint Density of States in One Dimension 98
- 8.14 Problem: One-dimensional Semiconductor 98
- 8.15 Problem: Conductivity of Graphene 99
- 8.16 Problem: Bloch Oscillations 99
- 9 Interactions and Phase Transitions 101**
- 9.1 Problem: Spontaneous Polarization 105
- 9.2 Problem: Divergent Susceptibility 106
- 9.3 Problem: Large- $U$  Hubbard Model 106
- 9.4 Problem: Infinite-range Hubbard Model 107
- 9.5 Problem: Stoner Model 107
- 9.6 Problem: One-dimensional Electron System 107
- 9.7 Problem: Peierls Distortion 108
- 9.8 Problem: Singularity at  $2k_F$  108
- 9.9 Problem: Susceptibility of a One-dimensional Electron Gas 109
- 9.10 Problem: Critical Temperature in Mean Field Approximation 109
- 9.11 Problem: Instability of Half-filled Band 109
- 9.12 Problem: Screening of an Impurity Charge 110
- 9.13 Problem: Fermi Surface Nesting in Two Dimensions 110
- 9.14 Problem: Fermi Surface Nesting in Quasi-one-dimension 111
- 9.15 Problem: Anderson Model 112
- 9.16 Problem: Collective Jahn–Teller Distortion 113

<b>10</b>	<b>Mesoscopic and Nanoscale Systems</b>	<b>115</b>
10.1	Problem: Differential Conductance of a Tunnel Junction	122
10.2	Problem: Sharvin Formula	122
10.3	Problem: Ballistic Transport	123
10.4	Problem: Tunneling from a Ballistic Conductor	123
10.5	Problem: Reflectivity of a Lattice Defect	124
10.6	Problem: Two Scatterers: Incoherent Propagation	125
10.7	Problem: Two Scatterers: Coherent Propagation	125
10.8	Problem: Strong Localization	126
10.9	Problem: Coulomb Blockade	126
10.10	Problem: Single-atom Transistor	127
<b>Part II</b>	<b>Solutions to Problems</b>	<b>129</b>
<b>11</b>	<b>Crystal Structures</b>	<b>131</b>
11.1	Solution: Symmetries	131
11.2	Solution: Rotations	131
11.3	Solution: Copper Oxide Layers	132
11.4	Solution: Graphite	133
11.5	Solution: Structure of $A_xC_{60}$	133
11.6	Solution: <i>hcp</i> and <i>fcc</i> Structures	134
11.7	Solution: <i>hcp</i> and <i>bcc</i> Structures	135
11.8	Solution: Structure Factor of $A_xC_{60}$	136
11.9	Solution: Neutron Diffraction Device	137
11.10	Solution: Linear Array of Emitters, Finite Size Effects	137
11.11	Solution: Linear Array of Emitters: Superlattice	139
11.12	Solution: Powder Diffraction of <i>hcp</i> and <i>fcc</i> Crystals	140
11.13	Solution: Momentum Resolution	141
11.14	Solution: Finite Size Effects	143
11.15	Solution: Random Displacement	145
11.16	Hint: Vacancies	146
11.17	Hint: Integrated Scattering Intensity	146
<b>12</b>	<b>Interatomic Forces and Lattice Vibrations</b>	<b>147</b>
12.1	Solution: Madelung Constant	147
12.2	Solution: NaCl Bulk Modulus	149
12.3	Hint: Madelung with Screened Potential	151
12.4	Solution: Triple-axis Spectrometer	151
12.5	Solution: Phonons in Silicon	151
12.6	Solution: Linear Array of Emitters: Phonons	152
12.7	Hint: Long-range Interaction	152
12.8	Solution: Mass Defect	153
12.9	Solution: Debye Frequency	154
12.10	Solution: Vibrations of a Square Lattice	155
12.11	Solution: Grüneisen Parameter	156

x | Contents

- 12.12 Solution: Diatomic Chain 157
- 12.13 Hint: Damped Oscillation 159
- 12.14 Solution: Two-dimensional Debye 159
- 12.15 Solution: Soft Optical Phonons 160
- 12.16 Hint: Soft Phonons Again 162
- 12.17 Solution: Nanowire Phonons 162
  
- 13 Electronic Band Structure 163**
- 13.1 Solution: Nearly Free Electrons in One Dimension 163
- 13.2 Solution: Eigenfunctions of Nearly Free Electrons in One Dimension 164
- 13.3 Solution: Tight-binding Hamiltonian 166
- 13.4 Solution: Nearly Free Electrons in Dirac-delta Potentials 166
- 13.5 Solution: Tight binding in Dirac-delta Potentials 166
- 13.6 Solution: Dirac-delta Potentials 168
- 13.7 Solution: Band Overlap 171
- 13.8 Solution: Nearly Free Electrons in Two Dimensions 172
- 13.9 Solution: Nearly Free-electron Bands 174
- 13.10 Solution: Instability at the Fermi Wavenumber 174
- 13.11 Solution: Electrons in 2D Nearly Free-electron Band 176
- 13.12 Solution: Square Lattice 178
- 13.13 Solution: Tight-binding Band in Two Dimensions 179
- 13.14 Solution: Electrons in 2D Tight-binding Band 180
- 13.15 Solution: Dirac-delta Potentials in Two Dimensions 181
- 13.16 Hint: Graphene 181
- 13.17 Hint: Effective Mass 182
- 13.18 Hint: Cyclotron Frequency 184
- 13.19 Solution: de Haas–van Alphen 184
- 13.20 Hint: Fermi Energy 184
  
- 14 Density of States 187**
- 14.1 Solution: Density of States 187
- 14.2 Solution: Two-dimensional Density of States 187
- 14.3 Solution: Two-dimensional Tight-binding 188
- 14.4 Solution: Tight-binding from One to Three Dimensions 189
- 14.5 Solution: Crossover to Quasi-one-dimensional Metal 191
- 14.6 Solution: Phonon Mode of Two-dimensional System 193
- 14.7 Solution: Saddle Point 194
- 14.8 Solution: Density of States in Superconductors 196
- 14.9 Solution: Energy Gap 197
- 14.10 Solution: Density of States for Hybridized Bands 197
- 14.11 Solution: Infinite-dimensional DOS 199
- 14.12 Solution: Two- to Three-dimensional Crossover 200
- 14.13 Solution: One- to Two-dimensional Crossover 201
- 14.14 Solution: Carbon Nanotube 202

<b>15</b>	<b>Elementary Excitations</b>	<b>203</b>
15.1	Solution: Tight-binding Model	203
15.2	Solution: Impurity State in the One-dimensional Tight-binding Model	205
15.3	Solution: Hybridization of Energy Bands	206
15.4	Solution: Polarons	208
15.5	Solution: Polaritons	210
15.6	Hint: Excitons	211
15.7	Solution: Holstein–Primakoff Transformation	212
15.8	Hint: Dyson–Maleev Representation	212
15.9	Solutions: Spin Waves	212
15.10	Solution: Spin Waves Again	214
15.11	Solution: Anisotropic Heisenberg Model	214
15.12	Hint: Solitons	215
15.13	Solution: Dirac Fermions in Graphene	216
<b>16</b>	<b>Thermodynamics of Noninteracting Quasiparticles</b>	<b>217</b>
16.1	Solution: Specific Heat of Metals and Insulators	217
16.2	Solution: Number of Phonons	217
16.3	Solution: Energy of the Phonon Gas	218
16.4	Solution: Bulk Modulus of the Phonon Gas	220
16.5	Hint: Phonons in One Dimension	221
16.6	Solution: Electron–Hole Symmetry	221
16.7	Solution: Entropy of the Noninteracting Electron Gas	223
16.8	Solution: Free Energy with Gap at the Fermi Energy	224
16.9	Solution: Bulk Modulus at $T = 0$	226
16.10	Solution: Temperature Dependence of the Bulk Modulus	226
16.11	Solution: Chemical Potential of the Free-electron Gas	227
16.12	Solution: EuO Specific Heat	229
16.13	Hint: Magnetization at Low Temperatures	230
16.14	Solution: Electronic Specific Heat	230
16.15	Solution: Quantum Hall Effect	232
<b>17</b>	<b>Transport Properties</b>	<b>235</b>
17.1	Solution: Temperature-dependent Resistance	235
17.2	Solution: Conductivity Tensor	236
17.3	Solution: Montgomery Method	238
17.4	Solution: Anisotropic Layer	238
17.5	Solution: Two-charge-carrier Drude Model	240
17.6	Solution: Thermal Conductivity	241
17.7	Hint: Residual Resistivity	243
17.8	Solution: Electric and Heat Transport	243
17.9	Solution: Conductivity of Tight-binding Band	244

17.10	Solution: Hall Effect in Two-dimensional Metals	246
17.11	Solution: Free-electron Results from the Boltzmann Equations	248
17.12	Solution: $p$ - $n$ Junctions	250
17.13	Solution: Mott–Ioffe–Regel	251
<b>18</b>	<b>Optical Properties</b>	<b>253</b>
18.1	Solution: Reflectivity and Transmission: Multiple Interfaces	253
18.2	Solution: Group Velocity	254
18.3	Solution: Tinkham Formula	255
18.4	Solution: Transmission of a Thin Superconductor	256
18.5	Solution: Fourier Transform Infrared Spectroscopy	257
18.6	Solution: Sum Rule: Dielectric Constant	257
18.7	Solution: Optical Mode of KBr	257
18.8	Solution: Direct-gap Semiconductor	258
18.9	Solution: Inversion Symmetry	259
18.10	Solution: Frequency-dependent Conductivity	260
18.11	Solution: Frequency-dependent Response of a Superconductor	263
18.12	Solution: Optical Conductivity Tensor	264
18.13	Solution: Joint Density of States in One Dimension	265
18.14	Solution: One-dimensional Semiconductor	266
18.15	Solution: Conductivity of Graphene	268
18.16	Solution: Bloch Oscillations	269
<b>19</b>	<b>Interactions and Phase Transitions</b>	<b>273</b>
19.1	Solution: Spontaneous Polarization	273
19.2	Solution: Divergent Susceptibility	274
19.3	Solution: Large- $U$ Hubbard Model	275
19.4	Solution: Infinite-range Hubbard Model	278
19.5	Hint: Stoner Model	279
19.6	Solution: One-dimensional Electron System	279
19.7	Solution: Peierls Distortion	281
19.8	Solution: Singularity at $2k_F$	282
19.9	Solution: Susceptibility of a One-dimensional Electron Gas	283
19.10	Solution: Critical Temperature in Mean Field Approximation	284
19.11	Solution: Instability of Half-filled Band	284
19.12	Solution: Screening of an Impurity Charge	285
19.13	Solution: Fermi Surface Nesting in Two Dimensions	287
19.14	Solution: Fermi Surface Nesting in Quasi One Dimension	288
19.15	Solution: Anderson Model	291
19.16	Solution: Collective Jahn–Teller Distortion	294
<b>20</b>	<b>Mesoscopic and Nano-scale Systems</b>	<b>297</b>
20.1	Solution: Differential Conductance of a Tunnel Junction	297
20.2	Solution: Sharvin Formula	297
20.3	Solution: Ballistic Transport	298

20.4	Solution: Tunneling from a Ballistic Conductor	299
20.5	Solution: Reflectivity of a Lattice Defect	299
20.6	Solution: Two Scatterers: Incoherent Propagation	300
20.7	Solution: Two Scatterers: Coherent Propagation	301
20.8	Solution: Strong Localization	301
20.9	Solution: Coulomb Blockade	302
20.10	Solution: Single-atom Transistor	303

**References** 305

**Index** 307

