A Creep and shrinkage

A1 Determination of creep and shrinkage values

1. Purpose of example

It is necessary to determine the values of creep and shrinkage of concrete in a composite beam with cross-section shown in Figure A1.1 as follows:

- The values of the creep coefficient at $t = \infty$, the final creep coefficient $\varphi(\infty, t_0)$, and at $t = 90$ days which is denoted with $\varphi(90, t_0)$,
- The values of the total shrinkage strain at $t = \infty$ which is denoted with $\varepsilon_{cs}(\infty)$ (the final value) and at $t = 90$ days which is denoted with $\varepsilon_{cs}(90)$.

2. Cross-section

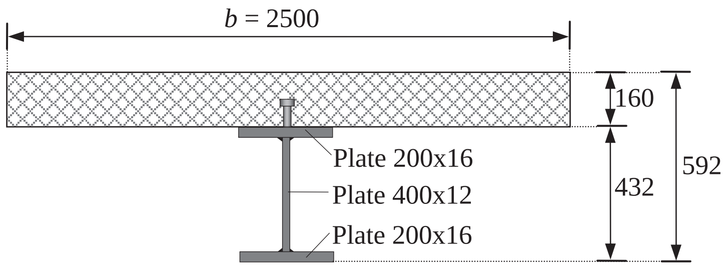

Figure A1.1 *Cross-section*

3. Input data

Concrete strength class: C 20/25 $f_{ck} = 20{,}0 \ \text{N/mm}^2$

$$f_{cd} = \frac{f_{ck}}{\gamma_c} = \frac{20{,}0}{1{,}5} = 13{,}3 \ \text{N/mm}^2$$

$$E_{cm} = 30000 \ \text{N/mm}^2$$

Type of cement: *N, strength class according to* EN 197-1, 32,5 R $\alpha = 0$

$$\alpha_{ds1} = 4$$

$$\alpha_{ds2} = 0{,}12$$

Relative humidity: *inside conditions* RH 50%

First loading $t_0 = 28$ days

Beginning of drying $t_s = 3$ days

Composite Structures according to Eurocode 4. Worked Examples.
First Edition. Darko Dujmović, Boris Androić, Ivan Lukačević.

4. Creep coefficients

4.1 Determination of final creep coefficient

For the calculation of the final creep coefficient $\varphi(\infty, t_0)$ the following is valid:

- the perimeter of that part which is exposed to drying, u

$$u = 2 \cdot b$$

$$u = 2 \cdot 2500 = 5000 \text{ mm}$$

- the notional size of the cross-section, h_0

$$h_0 = \frac{2 \cdot A_c}{u} = \frac{2 \cdot 2500 \cdot 160}{5000} = 160 \text{ mm } = 16 \text{ cm}$$

- t_0 = 28 days,
- inside conditions, the ambient relative humidity RH 50 %,
- the concrete strength class C 20/25,
- the type of cement – cement class N, strength class 32,5 R.

The final value of creep coefficient $\varphi(\infty, t_0)$ is determined using the nomogram shown in Figure 3.1, EN 1992-1-1. The process of determining the final value of the creep coefficient, taking into account these assumptions, is given in Figure A1.2:

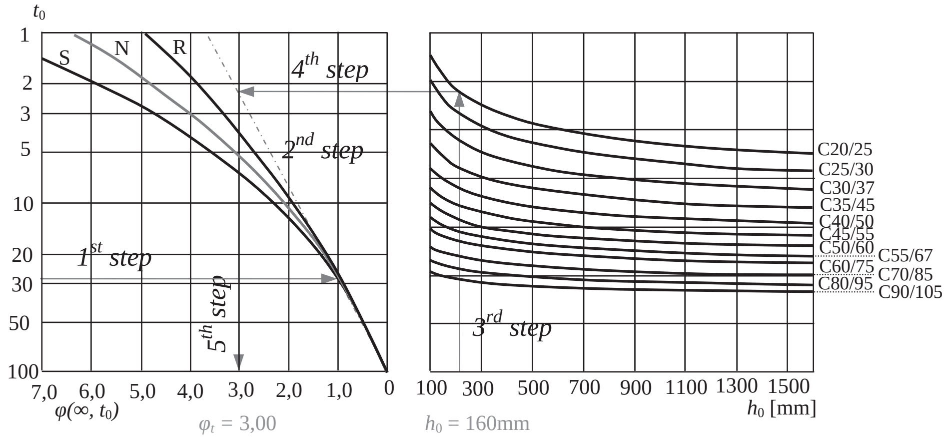

Figure A1.2 *Method for determining the creep coefficient*

The value of the final creep coefficient found from Figure A1.2 is:

$$\varphi_t = \varphi(\infty, t_0) = 3,00$$

4.2 Determination of creep coefficient at time t = 90 days

The value of creep coefficient $\varphi(t, t_0)$ for some arbitrary time t can be calculated from:

$$\varphi(t,t_0) = \varphi_0 \cdot \beta_c(t,t_0)$$

where:

φ_0 is the notional creep coefficient,

$\beta_c(t, t_0)$ is a coefficient to describe the development of creep with time after loading (at $t_0 = 0$, $\beta_c(t, t_0) = 0$, and at $t = \infty$, $\beta_c(t, t_0) = 1$),

The value of φ_0 is obtained as:

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$$

where:

φ_{RH} is a factor to allow for the effect of relative humidity on the notional creep coefficient and is calculated as follows:

$$\varphi_{RH} = 1 + \frac{1 - RH/100}{0{,}1 \cdot \sqrt[3]{h_0}} \quad \text{for } f_{cm} \le 35 \text{ N/mm}^2$$

$$\varphi_{RH} = [1 + \frac{1 - RH/100}{0{,}1 \cdot \sqrt[3]{h_0}} \cdot \alpha_1] \cdot \alpha_2 \quad \text{for } f_{cm} > 35 \text{ N/mm}^2$$

RH is the relative humidity of the ambient environment (in %),

h_0 is the notional size of the cross-section of the member (h_0 in mm), $h_0 = 2A_c / u$

A_c is the cross-sectional area of concrete (mm^2),

u is the perimeter of the member in contact with the atmosphere (mm),

$\beta(f_{cm})$ is a factor to allow for the effect of concrete strength on the notional creep coefficient and is determined as follows:

$$\beta(f_{cm}) = \frac{16{,}8}{\sqrt{f_{cm}}}$$

where:

f_{cm} is the mean compressive cylinder strength of concrete at the age of 28 days (N/mm^2, $f_{cm} = f_{ck}$ + 8 N/mm^2),

$\beta(t_0)$ is a factor to allow for the effect of concrete age at loading on the notional creep coefficient and is determined as follows:

$$\beta(t_0) = \frac{1}{(0{,}1 + t_0^{0{,}20})}$$

The effect of the type of cement on the creep coefficient of concrete can be taken into account by modifying the age of loading t_0 according to the following expression:

$$t_0 = t_{0,T} \cdot [\frac{9}{2 + t_{0,T}^{\;1{,}2}} + 1]^{\alpha} \geq 0{,}5 \text{ days}$$

where:

α is the factor that takes into account the development of concrete strength as a function of type of cement,

$t_{0,T}$ is the temperature-adjusted age of concrete at loading in days.

The effect of elevated or reduced temperatures within the range 0–80°C on the maturity of concrete can be taken into account by adjusting the concrete age according to the following expression:

$$t_T = \sum_{i=1}^{n} e^{-(4000/[273+T(\Delta t_i)]-13{,}65])} \cdot \Delta t_i$$

where:

t_T is the temperature-adjusted concrete age which replaces t in the corresponding expressions,

$T(\Delta t_i)$ is the temperature in °C during the time period Δt_i,

Δt_i is the number of days where a temperature T prevails.

$$\beta_c(t, t_0) = [\frac{(t - t_0)}{\beta_H + (t - t_0)}]^{0{,}3}$$

where:

t is the age of concrete in days at the time considered (in days),

t_0 is the age of concrete at first loading (in days),

$t - t_0$ is the non-adjusted duration of loading in days,

β_H is the coefficient depending on the relative humidity (RH in %) and the notional member size (h_0 in mm), and is estimated according to expressions:

$$\beta_H = 1{,}5 \cdot [1 + (0{,}012 \cdot RH)^{18}] \cdot h_0 + 250 \leq 1500 \quad \text{for } f_{cm} \leq 35 \text{ N/mm}^2$$

$$\beta_H = 1{,}5 \cdot [1 + (0{,}012 \cdot RH)^{18}] \cdot h_0 + 250 \cdot \alpha_3 \leq 1500 \cdot \alpha_3 \quad \text{for } f_{cm} > 35 \text{ N/mm}^2$$

α_i are correction factors which take into account the influence of the concrete strength according to the following expressions:

$$\alpha_1 = [35 / f_{cm}]^{0,7}$$

$$\alpha_2 = [35 / f_{cm}]^{0,2}$$

$$\alpha_3 = [35 / f_{cm}]^{0,5}$$

Thus, the mean compressive cylinder strength of concrete from Table 3.1, EN 1992-1-1 is:

$$f_{cm} = f_{ck} + 8 \ \text{N/mm}^2 = 20 + 8 = 28 \ \ \text{N/mm}^2$$

$$\alpha = 0 \ \text{(type of cement N)}$$

Correction factors which taken into account the influence of the concrete strength are:

$$\alpha_1 = [35 / f_{cm}]^{0,7} = [35 / 28]^{0,7} = 1{,}17$$

$$\alpha_2 = [35 / f_{cm}]^{0,2} = [35 / 28]^{0,2} = 1{,}05$$

$$\alpha_3 = [35 / f_{cm}]^{0,5} = [35 / 28]^{0,5} = 1{,}12$$

The factor to allow for the effect of relative humidity on the notional creep coefficient φ_0 for $f_{cm} \leq 35$ N/mm^2 is:

$$\varphi_{RH} = 1 + \frac{1 - RH / 100}{0{,}1 \cdot \sqrt[3]{h_0}} = 1 + \frac{1 - 50 / 100}{0{,}1 \cdot \sqrt[3]{160}} = 1{,}92$$

The factor to allow for the effect of concrete strength on the notional creep coefficient φ_0 is:

$$\beta(f_{cm}) = \frac{16{,}8}{\sqrt{f_{cm}}} = \frac{16{,}8}{\sqrt{28}} = 3{,}18$$

The effect of the type of cement on the creep coefficient of concrete can be taken into account by modifying the age of loading t_0 according to the following expression, where $t_{0,T} = t_0 = 28$ days:

$$t_0 = t_{0,T} \cdot [\frac{9}{2 + t_{0,T}{}^{1,2}} + 1]^{\alpha} = 28 \cdot [\frac{9}{2 + 28^{1,2}} + 1]^{0}$$

$$t_0 = 28 \text{ days} \geq 0,5 \text{ days}$$

The factor to allow for the effect of concrete age at loading on the notional creep coefficient φ_0 is:

$$\beta(t_0) = \frac{1}{(0,1 + t_0^{0,20})} = \frac{1}{(0,1 + 28^{0,2})} = 0,49$$

The coefficient depending on the relative humidity (RH in %) and the notional member size h_0 for $f_{cm} \leq 35$ N/mm^2 is:

$$\beta_H = 1,5 \cdot [1 + (0,012 \cdot RH)^{18}] \cdot h_0 + 250 =$$
$$= 1,5 \cdot [1 + (0,012 \cdot 50)^{18}] \cdot 160 + 250 = 490 \leq 1500$$

The coefficient to describe the development of creep with time after loading is:

$$\beta_c(t, t_0) = [\frac{(t - t_0)}{\beta_H + (t - t_0)}]^{0,3} = [\frac{(90 - 28)}{490 + (90 - 28)}]^{0,3} = 0,52$$

The notional creep coefficient φ_0 is:

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0) = 1,92 \cdot 3,18 \cdot 0,49 = 2,99 \approx 3,0$$

The value of notional creep coefficient represents the value of the final creep coefficient $\varphi(\infty, t_0)$ found from Figure A1.2. Thus, this result confirms the accuracy of the results obtained from the Figure A1.2 − see Section 4.1.

At t = 90 days the creep coefficient $\varphi(t, t_0)$ is:

$$\varphi(t, t_0) = \varphi_0 \cdot \beta_c(t, t_0) = 2,99 \cdot 0,52 = 1,55$$

5. Shrinkage strains

5.1 Determination of final value of shrinkage strain

The total shrinkage strain of concrete, ε_{cs}, is composed of two components:

$\varepsilon_{cs}(\infty) = \varepsilon_{cd}(\infty) + \varepsilon_{ca}(\infty)$

where:

$\varepsilon_{cd}(\infty)$ is the drying shrinkage strain,
ε_{ca} is the autogenous shrinkage strain (this develops during hardening of the concrete).

The final value of the drying shrinkage strain $\varepsilon_{cd}(\infty)$ is:

$$\varepsilon_{cd}(\infty) = k_h \cdot \varepsilon_{cd,0}$$

where:

k_h is a coefficient depending on the notional size of the member h_0, Table A1.1,
$\varepsilon_{cd,0}$ is the nominal unrestrained drying shrinkage value, which can be taken from Table A1.2 or can be calculated by means of the following expression:

$$\varepsilon_{cd,0} = 0{,}85 \cdot [(220 + 110 \cdot \alpha_{ds1}) \cdot \exp(-\alpha_{ds2} \cdot \frac{f_{cm}}{10})] \cdot 10^{-6} \cdot \beta_{RH}$$

$$\beta_{RH}(RH) = 1{,}55 \cdot [1 - (\frac{RH}{100})^3]$$

where:

f_{cm} is the mean compressive cylinder strength of concrete at the age of 28 days (N/mm^2, $f_{cm} = f_{ck}$ + 8 N/mm^2),
α_{dsi} are factors which depend on the type of cement,
RH is the ambient relative humidity (%).

Table A1.1 *Values for factor k_h for calculation of final value of drying shrinkage strain*

h_0 [mm]	k_h
100	1,00
200	0,85
300	0,75
≥ 500	0,70

(1) h_0 – notional size of member (mm)

(2) h_0 = 2 x (cross-sectional area of concrete A_c)/(perimeter of member in contact with atmosphere)

Table A1.2 *Nominal unrestrained drying shrinkage values of $\varepsilon_{cd,0}$ (in ‰) for concrete with cement class N*

	Relative Humidity	
$f_{ck,cy}/f_{ck,cube}$ (N/mm²)	Inside conditions, 50%	Outside conditions, 80%
20/25	0,54	0,30
40/50	0,42	0,24
60/75	0,33	0,19
80/95	0,26	0,15
90/105	0,23	0,13

The final value of the drying shrinkage strain $\varepsilon_{cd}(\infty)$ are determined using Tables A1.1 and A1.2.

The nominal unrestrained drying shrinkage value $\varepsilon_{cd,0}$ according to Table A1.2 for concrete strength class C 20/25 and RH 50% is 0,54‰.

The factor k_h depending on the notional size of the member h_0 according to Table A1.1 is:

For h_0 = 100 mm $\rightarrow$ k_h = 1,0
For h_0 = 200 mm $\rightarrow$ k_h = 0,85

Linear interpolation:

For h_0 = 160 mm $\rightarrow$ $k_h = 0{,}85 + \dfrac{200-160}{200-100} \cdot (1{,}0 - 0{,}85)$

k_h = 0,91

The final value of the drying shrinkage strain is:

$$\varepsilon_{cd}(\infty) = k_h \cdot \varepsilon_{cd,0} = 0{,}91 \cdot 5{,}4 \cdot 10^{-4} = 4{,}91 \cdot 10^{-4}$$

The final value of the autogenous shrinkage strain is:

$$\varepsilon_{ca}(\infty) = 2{,}5 \cdot (f_{ck} - 10) \cdot 10^{-6} = 2{,}5 \cdot (25 - 10) \cdot 10^{-6} = 3{,}75 \cdot 10^{-5}$$

The total shrinkage strain $\varepsilon_{cs}(\infty)$ is:

$$\varepsilon_{cs}(\infty) = \varepsilon_{cd}(\infty) + \varepsilon_{ca}(\infty) = 4{,}91 \cdot 10^{-4} + 3{,}75 \cdot 10^{-5} = 5{,}29 \cdot 10^{-4}$$

$$\varepsilon_{cs}(\infty) = 0{,}529 \text{ ‰}$$

5.2 Determination of shrinkage strain at time *t* = 90 days

The total shrinkage strain at time *t* is calculated as:

$$\varepsilon_{cs}(t) = \varepsilon_{cd}(t) + \varepsilon_{ca}(t)$$

The value of the drying shrinkage strain ε_{cd} at time *t* is:

$$\varepsilon_{cd}(t) = \beta_{ds}(t,t_s) \cdot k_h \cdot \varepsilon_{cd,0}$$

where:

$$\beta_{ds}(t,t_s) = \frac{(t-t_s)}{(t-t_s)+0,04\sqrt{h_0^3}}$$

t is the age of the concrete at the time considered, in days,

t_s is the age of the concrete in days at the beginning of drying shrinkage; normally this is at the end of the curing of the concrete.

The value of the autogenous shrinkage strain ε_{ca} at the age of concrete *t*, is given with the following expression:

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty)$$

where:

$$\beta_{as}(t) = 1 - \exp(-0,2\sqrt{t})\text{, } t \text{ in days}$$

$$\varepsilon_{ca}(\infty) = 2,5 \cdot (f_{ck} - 10) \cdot 10^{-6}$$

The drying shrinkage strain is:

$$\varepsilon_{cd}(t) = \beta_{ds}(t,t_s) \cdot k_h \cdot \varepsilon_{cd,0}$$

$$\beta_{ds}(t,t_s) = \frac{(t-t_s)}{(t-t_s)+0,04\sqrt{h_0^3}} = \frac{(90-3)}{(90-3)+0,04\sqrt{160^3}} = 0,52$$

From Section 5.1 k_h = 0,91.

$$\alpha_{ds1} = 4 \quad \alpha_{ds2} = 0,12$$

$$\varepsilon_{cd,0} = 0,85[(220 + 110 \cdot \alpha_{ds1}) \cdot \exp(-\alpha_{ds2} \cdot f_{cm} / 10)] \cdot 10^{-6} \cdot \beta_{RH}$$
$$= 0,85[(220 + 110 \cdot 4) \cdot \exp(-0,12 \cdot 28 / 10)] \cdot 10^{-6} \cdot 1,36 = 5,45 \cdot 10^{-4}$$

$$\beta_{RH}(RH) = 1,55 \cdot [1 - (RH / 100)^3] = 1,55 \cdot [1 - (50 / 100)^3] = 1,36$$

$$\varepsilon_{cd}(t) = \beta_{ds}(t, t_s) \cdot k_h \cdot \varepsilon_{cd,0} = 0,52 \cdot 0,91 \cdot 5,45 \cdot 10^{-4} = 2,58 \cdot 10^{-4}$$

The autogenous shrinkage strain is:

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty)$$

$$\varepsilon_{ca}(\infty) = 2,5 \cdot (f_{ck} - 10) \cdot 10^{-6} = 2,5 \cdot (25 - 10) \cdot 10^{-6} = 3,75 \cdot 10^{-5}$$

$$\beta_{as}(t) = 1 - \exp(-0,2\sqrt{t}) = 1 - \exp(-0,2\sqrt{90}) = 0,85$$

$$\varepsilon_{ca}(t) = \beta_{as}(t) \cdot \varepsilon_{ca}(\infty) = 0,85 \cdot 3,75 \cdot 10^{-5} = 3,19 \cdot 10^{-5}$$

The total shrinkage strain is:

$$\varepsilon_{cs}(t) = \varepsilon_{cd}(t) + \varepsilon_{ca}(t)$$

$$\varepsilon_{cs}(90) = 2,58 \cdot 10^{-4} + 3,19 \cdot 10^{-5} = 2,89 \cdot 10^{-4} = 0,289 ‰$$

6. Commentary

The effects of time-dependent strains of concrete (creep and shrinkage) in the analysis of structural elements of composite structures are different according to whether they are observed in *the level of cross-section or static system.*

The effects of creep and shrinkage of concrete produce internal forces and moments in cross-sections, and curvatures and longitudinal strains in members. The effects that occur in statically determinate systems are classified as primary effects. In statically indeterminate system, the primary effects of creep and shrinkage are associated with additional action effects, such that the total effects are compatible. These are classified as secondary effects and are considered as indirect actions which are sets of imposed deformations.

Computational methods, principles and basic equations for estimating the time-dependent strains of concrete are given in EN 1992-1-1. The determination of the final value of the creep coefficient $\varphi(\infty, t_0)$ for concrete under normal environmental conditions is possible using nomograms. However, for the determination of the values of shrinkage strains we need to use the extensive numerical procedure.